CS/Math 514, Fall 2019 – Numerical Analysis (Roos)
Homework assignment 3.
Due Monday, September 30.

1. Do Problems 8.1, 8.8, 8.12 in the textbook.

2. The nth Chebyshev polynomial T_n is defined as the unique polynomial such that $T_n(x) = \cos(n\theta)$ where θ is such that $\cos(\theta) = x$.
(i) Let a_n denote the sum of the absolute values of the coefficients of the polynomial T_n. For example, $a_2 = 3$ and $a_{10} = 3363$. By writing an appropriate program, compute a_{514}.
(ii) Compute the sum of the coefficients (not their absolute values) of $T_{10^{20}}$.

3. Let $\delta \in (0, 1)$. Compute the minimax polynomial of degree one to the function $f(x) = \sqrt{1 + x^2}$ on $[0, \delta]$. What happens as δ approaches zero?
Hint: Read the discussion after Thm. 8.5 in the textbook.

Honors Problem 1. Consider the function $f(x) = \sin(\pi x)$. We would like to approximate f by a polynomial of degree n on the interval $[0, 1]$. Compute and plot each of the following types of approximating polynomials for degrees $n = 2, 5, 8$:

(i) Bernstein polynomials (as in Exercise 8.12)
(ii) Lagrange polynomial with equidistant points (with $x_k = \frac{k}{n}$, $k = 0, \dots, n$)
(iii) Lagrange polynomial with Chebyshev points (as in Theorem 8.7)
(iv) Taylor polynomial at $x = \frac{1}{2}$

Also determine the maximum error in each case. Discuss the results.