1. This is false, as the dot product of two vectors is a scalar, not a vector.

2. False. For example, if $\mathbf{u} = \mathbf{i}$ and $\mathbf{v} = -\mathbf{i}$ then $|\mathbf{u} + \mathbf{v}| = |\mathbf{0}| = 0$ but $|\mathbf{u}| + |\mathbf{v}| = 1 + 1 = 2$.

3. False. For example, if $\mathbf{u} = \mathbf{i}$ and $\mathbf{v} = \mathbf{j}$ then $|\mathbf{u} \cdot \mathbf{v}| = |0| = 0$ but $|\mathbf{u}|\,|\mathbf{v}| = 1 \cdot 1 = 1$. In fact, by Theorem 10.3.3, $|\mathbf{u} \cdot \mathbf{v}| = \big||\mathbf{u}|\,|\mathbf{v}| \cos\theta\big|$.

4. False. For example, $|\mathbf{i} \times \mathbf{i}| = |\mathbf{0}| = 0$ (see Example 10.4.2) but $|\mathbf{i}|\,|\mathbf{i}| = 1 \cdot 1 = 1$. In fact, by Theorem 10.4.6, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin\theta$.

5. True, by Property 2 of the dot product. (See page 551.)

6. False. Theorem 10.4.8 says that $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$. (See page 562.)

7. True. If θ is the angle between $\mathbf{u}$ and $\mathbf{v}$, then by Theorem 10.4.6, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin\theta = |\mathbf{v}|\,|\mathbf{u}| \sin\theta = |\mathbf{v} \times \mathbf{u}|$. (Or, by Theorem 10.4.8, $|\mathbf{u} \times \mathbf{v}| = |-\mathbf{v} \times \mathbf{u}| = |-1|\,|\mathbf{v} \times \mathbf{u}| = |\mathbf{v} \times \mathbf{u}|$.)

8. This is true by Property 4 of the dot product.

9. Property 2 of the cross product tells us that this is true.

10. This is true by Theorem 10.4.8.

11. This is true by Theorem 10.4.8.

12. In general, this assertion is false; a counterexample is $\mathbf{i} \times (\mathbf{i} \times \mathbf{j}) \neq (\mathbf{i} \times \mathbf{i}) \times \mathbf{j}$. (See the paragraph preceding Theorem 10.4.8.)

13. This is true because $\mathbf{u} \times \mathbf{v}$ is orthogonal to $\mathbf{u}$ (see Theorem 10.4.5), and the dot product of two orthogonal vectors is 0.

14. $$\begin{aligned}(\mathbf{u} + \mathbf{v}) \times \mathbf{v} &= \mathbf{u} \times \mathbf{v} + \mathbf{v} \times \mathbf{v} && \text{[by Theorem 10.4.8]}\\ &= \mathbf{u} \times \mathbf{v} + \mathbf{0} && \text{[by Example 10.4.2]}\\ &= \mathbf{u} \times \mathbf{v}, \text{ so this is true.}\end{aligned}$$

15. This is false. A normal vector to the plane is $\mathbf{n} = \langle 6, -2, 4\rangle$. Because $\langle 3, -1, 2\rangle = \frac{1}{2}\mathbf{n}$, the vector is parallel to $\mathbf{n}$ and hence perpendicular to the plane.

16. This is false, because according to Equation 10.5.8, $ax + by + cz + d = 0$ is the general equation of a plane.

17. This is false. In $\mathbb{R}^2$, $x^2 + y^2 = 1$ represents a circle, but $\{(x, y, z) \mid x^2 + y^2 = 1\}$ represents a *three-dimensional surface*, namely, a circular cylinder with axis the z-axis.

18. This is false. In $\mathbb{R}^3$ the graph of $y = x^2$ is a parabolic cylinder (see Example 10.6.1). A paraboloid has an equation such as $z = x^2 + y^2$.

19. False. For example, $\mathbf{i} \cdot \mathbf{j} = \mathbf{0}$ but $\mathbf{i} \neq \mathbf{0}$ and $\mathbf{j} \neq \mathbf{0}$.

20. This is false. By Corollary 10.4.7, $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ for any nonzero parallel vectors $\mathbf{u}$, $\mathbf{v}$. For instance, $\mathbf{i} \times \mathbf{i} = \mathbf{0}$.

21. This is true. If $\mathbf{u}$ and $\mathbf{v}$ are both nonzero, then by in , $\mathbf{u} \cdot \mathbf{v} = 0$ implies that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. But $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ implies that $\mathbf{u}$ and $\mathbf{v}$ are parallel (see). Two nonzero vectors can't be both parallel and orthogonal, so at least one of $\mathbf{u}$, $\mathbf{v}$ must be $\mathbf{0}$.

22. This is true. We know $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\ |\mathbf{v}|\ \cos\theta$ where $|\mathbf{u}| \geq 0$, $|\mathbf{v}| \geq 0$, and $|\cos\theta| \leq 1$, so $|\mathbf{u} \cdot \mathbf{v}| = |\mathbf{u}|\ |\mathbf{v}|\ |\cos\theta| \leq |\mathbf{u}|\ |\mathbf{v}|$.

23. True. If we reparametrize the curve by replacing $u = t^3$, we have $\mathbf{r}(u) = u\,\mathbf{i} + 2u\,\mathbf{j} + 3u\,\mathbf{k}$, which is a line through the origin with direction vector $\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$.

24. True. Parametric equations for the curve are $x = 0$, $y = t^2$, $z = 4t$, and since $t = z/4$ we have $y = t^2 = (z/4)^2$ or $y = \frac{1}{16}z^2$, $x = 0$. This is an equation of a parabola in the yz-plane.

25. False. The vector function represents a line, but the line does not pass through the origin; the x-component is 0 only for $t = 0$ which corresponds to the point $(0, 3, 0)$ not $(0, 0, 0)$.

26. True. See Theorem 10.7.4.

27. False. By Formula 5 of Theorem 10.7.5, $\dfrac{d}{dt}\,[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$.

28. False. For example, let $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$. Then $|\mathbf{r}(t)| = \sqrt{\cos^2 t + \sin^2 t} = 1 \quad \Rightarrow \quad \dfrac{d}{dt}\,|\mathbf{r}(t)| = 0$, but $|\mathbf{r}'(t)| = |\langle -\sin t, \cos t \rangle| = \sqrt{(-\sin t)^2 + \cos^2 t} = 1$.

29. False. κ is the magnitude of the rate of change of the unit tangent vector $\mathbf{T}$ with respect to arc length s, not with respect to t.

30. False. The binormal vector, by the definition given in Section 10.8, is $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t) = -\,[\mathbf{N}(t) \times \mathbf{T}(t)]$.

31. True. At an inflection point where f is twice continuously differentiable we must have $f''(x) = 0$, and by Equation 10.8.11, the curvature is 0 there.

32. True. From Equation 10.8.9, $\kappa(t) = 0 \quad \Leftrightarrow \quad |\mathbf{T}'(t)| = 0 \quad \Leftrightarrow \quad \mathbf{T}'(t) = \mathbf{0}$ for all t. But then $\mathbf{T}(t) = \mathbf{C}$, a constant vector, which is true only for a straight line.

33. False. If $\mathbf{r}(t)$ is the position of a moving particle at time t and $|\mathbf{r}(t)| = 1$ then the particle lies on the unit circle or the unit sphere, but this does not mean that the speed $|\mathbf{r}'(t)|$ must be constant. As a counterexample, let $\mathbf{r}(t) = \langle t, \sqrt{1 - t^2} \rangle$, then $\mathbf{r}'(t) = \langle 1, -t/\sqrt{1 - t^2} \rangle$ and $|\mathbf{r}(t)| = \sqrt{t^2 + 1 - t^2} = 1$ but $|\mathbf{r}'(t)| = \sqrt{1 + t^2/(1 - t^2)} = 1/\sqrt{1 - t^2}$ which is not constant.

34. True. See Example 11 in Section 10.7.