Math 141, Problem Set #1
(due **in class** Fri.. 9/13/13)

Stewart, section 1.1, problems 6, 8, 16, 20, 26, 28, 30, 34, 38, and 48. (For problem 38, please give a two-part definition of the function $g(x)$, asserting that $g(x)$ equals one linear function of x for $x \geq 0$ and equals a different linear function of x for $x < 0$. For problem 48, don't forget the instructions that govern problems 47–51.)

Also do the following problems. Note that for problems D and E, define "The function f is increasing on the set S" to mean "For all $x_1 < x_2$ in S, $f(x_1) < f(x_2)$".

A. Let x, y, and z respectively denote Alice's score on the homework, midterm, and final exam for Math 141, so that her score S for the course as a whole is determined by the formula

$$S = \max(0.3x + 0.3y + 0.4z, 0.3x + 0.4y + 0.3z, 0.4x + 0.3y + 0.3z).$$

(Note: $\max(a, b, c)$ is equal to whichever of the three numbers a, b, c is largest.) Suppose x is 70, y is 80, and z is unknown. Draw the graph for S as a function of z, where z ranges from 0 to 100, and give a piecewise definition of the function.

B. Sketch the curve given by $f(x) = x/x$.

C. Find a function $f(x)$ (defined for all values of x) that is both even and odd, or prove that such a function does not exist.

D. Suppose $f(x)$ is increasing on $\{x : x < 0\}$ and increasing on $\{x : x > 0\}$. Must $f(x)$ be increasing on $\{x : x \neq 0\}$? Prove or disprove.

E. Suppose $f(x)$ is increasing on $[0, 1]$ and increasing on $[1, 2]$. Must $f(x)$ be increasing on $[0, 2]$? Prove or disprove.

F. Suppose we know three things about the function f: f is increasing on the interval $I = [1, 3]$, $f(1) = 10$, and $f(3) = 20$. Which values of $f(2)$ are consistent with this information, and which are inconsistent with this information? Be sure to be explicit about the boundary cases $f(2) = 10$ and $f(2) = 20$; are they consistent, or inconsistent, with the other information?

G. Suppose we know two things about the function f: f is increasing on the interval $[0, \infty)$, and $f(x)$ is an even function of x (for all real numbers x). Prove that f is decreasing on the interval $(-\infty, 0]$.

Be explicit about your reasoning! For instance, instead of saying something "obviously isn't a function", say "it fails the vertical line test at $x = 1$".

Please don't forget to write down **who you worked on the assignment with** (if nobody, then write "I worked alone"), and record **how much time you spent on each problem** (this doesn't need to be exact) on the time-sheets I gave out in class.