Math 141, Problem Set #4
(due **in class** Fri., 10/4/13)

Note: To get full credit for a non-routine problem, it is not enough to give the right answer; you must explain your reasoning.

Stewart, section 1.5, problems 4, 14, 18, 30, 32, 36, 50, 51.

- Clarification for problem 36: You must restrict yourself to functions f such that $f(x)$ is defined for all x in $[0, 1]$; no fair leaving $f(0.25)$ undefined!

- First hint for problem 51: Define $u(t)$ to be the monk's distance from the monastery t seconds after midnight on the first day, and define $d(t)$ to be his distance from the monastery t seconds after midnight on the second day. Plot the functions $u(t)$ and $d(t)$ against the same coordinate axes. Saying that the monk is at the same point on his path at time t_0 on both days is equivalent to saying that $u(t_0) = d(t_0)$, i.e., the graphs of the functions $u(t)$ and $d(t)$ intersect at $t = t_0$.

- Second hint for problem 51: Showing that there exists a time t_0 such that $u(t_0) = d(t_0)$ is equivalent to showing that there exists a time t_0 such that the function $u(t)-d(t)$ takes the value 0 at $t = t_0$. Prove this by applying the Intermediate Value Theorem to the function $u(t)-d(t)$.

Also:

A. Suppose $f(x)$ is continuous at $x = a$ and $g(x)$ is discontinuous at $x = a$. Either prove that $(f+g)(x)$ must be discontinuous at $x = a$ or give an example to show that it is possible for $(f + g)(x)$ to be continuous at $x = a$.

B. Suppose $f(x)$ is continuous at $x = a$ and $g(x)$ is discontinuous at $x = a$. Either prove that $(fg)(x)$ must be discontinuous at $x = a$ or give an example to show that it is possible for $(fg)(x)$ to be continuous at $x = a$.

C. Stewart's statement of the Intermediate Value Theorem contains the requirement that f is continuous on the closed interval $[a, b]$. Show by a well-chosen example that this requirement cannot be replaced by the weaker requirement that f is defined on the closed interval $[a, b]$ and continuous on the open interval (a, b).

Note that if you're proving something, you'd better either refer to the definitions of the terms that occur in the statement you're proving or refer to theorems that have been proved about those concepts. For instance, if you're trying to prove that something is continuous, you should either directly verify that the definition of continuity is satisfied (which means verifying that a certain limit exists and equals a certain value) or make use of specific theorems (in the book or covered in class) of the form "If [blah blah blah] then f is continuous (being sure to verify that [blah blah blah] is true!).

Please don't forget to write down on your assignment **who you worked on the assignment with** (if nobody, then write "I worked alone"), and write down on your time-sheet **how many minutes you spent on each problem** (this doesn't need to be exact).