
True-False Quiz for Chapter 5

1. True by Property 2 of the Integral in Section 5.2.
2. False. Try $a = 0$, $b = 2$, $f(x) = g(x) = 1$ as a counterexample.
3. True by Property 3 of the Integral in Section 5.2.
4. False. You can't take a variable outside the integral sign. For example, using $f(x) = 1$ on $[0, 1]$,
$$\int_0^1 x f(x) dx = \int_0^1 x dx = \left[\frac{1}{2}x^2\right]_0^1 = \frac{1}{2}$$
 (a constant) while $x \int_0^1 1 dx = x [x]_0^1 = x \cdot 1 = x$ (a variable).
5. False. For example, let $f(x) = x^2$. Then $\int_0^1 \sqrt{x^2} dx = \int_0^1 x dx = \frac{1}{2}$, but $\sqrt{\int_0^1 x^2 dx} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$.
6. True by the Net Change Theorem.
7. True by Comparison Property 7 of the Integral in Section 5.2.
8. False. For example, let $a = 0$, $b = 1$, $f(x) = 3$, $g(x) = x$. $f(x) > g(x)$ for each x in $(0, 1)$, but $f'(x) = 0 < 1 = g'(x)$ for $x \in (0, 1)$.
9. True. The integrand is an odd function that is continuous on $[-1, 1]$, so the result follows from Theorem 5.5.7(b).
10. True.
$$\begin{aligned}\int_{-5}^5 (ax^2 + bx + c) dx &= \int_{-5}^5 (ax^2 + c) dx + \int_{-5}^5 bx dx \\ &= 2 \int_0^5 (ax^2 + c) dx \text{ [by 5.5.7(a)]} + 0 \text{ [by 5.5.7(b)]}\end{aligned}$$
11. False. For example, the function $y = |x|$ is continuous on \mathbb{R} , but has no derivative at $x = 0$.
12. True by FTC1.
13. True by Property 5 of Integrals.
14. False. For example, $\int_0^1 (x - \frac{1}{2}) dx = [\frac{1}{2}x^2 - \frac{1}{2}x]_0^1 = (\frac{1}{2} - \frac{1}{2}) - (0 - 0) = 0$, but $f(x) = x - \frac{1}{2} \neq 0$.
15. False. $\int_a^b f(x) dx$ is a constant, so $\frac{d}{dx} \left(\int_a^b f(x) dx \right) = 0$, not $f(x)$ [unless $f(x) = 0$]. Compare the given statement carefully with FTC1, in which the upper limit in the integral is x .
16. False. See Figure 6 and the remarks before it in Section 5.2, and notice that $y = x - x^3 < 0$ for $1 < x \leq 2$.
17. False. The function $f(x) = 1/x^4$ is not bounded on the interval $[-2, 1]$. It has an infinite discontinuity at $x = 0$, so it is not integrable on the interval. (If the integral were to exist, a positive value would be expected, by Comparison Property 6 of Integrals.)

18. False. For example, if $f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } -1 \leq x < 0 \end{cases}$ then f has a jump discontinuity at 0, but $\int_{-1}^1 f(x) dx$ exists and is equal to 1.