1. False. Since the numerator has a higher degree than the denominator, $\dfrac{x(x^2+4)}{x^2-4} = x + \dfrac{8x}{x^2-4} = x + \dfrac{A}{x+2} + \dfrac{B}{x-2}$.

2. True. In fact, $A = -1$, $B = C = 1$.

3. False. It can be put in the form $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-4}$.

4. False. The form is $\dfrac{A}{x} + \dfrac{Bx+C}{x^2+4}$.

5. False. This is an improper integral, since the denominator vanishes at $x = 1$.

$$\int_0^4 \frac{x}{x^2-1}\,dx = \int_0^1 \frac{x}{x^2-1}\,dx + \int_1^4 \frac{x}{x^2-1}\,dx \text{ and}$$

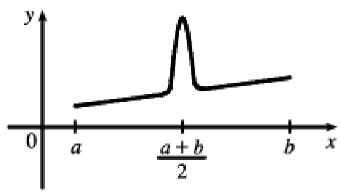
$$\int_0^1 \frac{x}{x^2-1}\,dx = \lim_{t\to 1^-} \int_0^t \frac{x}{x^2-1}\,dx = \lim_{t\to 1^-} \left[\tfrac{1}{2}\ln\left|x^2-1\right|\right]_0^t = \lim_{t\to 1^-} \tfrac{1}{2}\ln\left|t^2-1\right| = \infty$$

So the integral diverges.

6. True by Theorem 6.6.2 with $p = \sqrt{2} > 1$.

7. False. See Exercise 51 in Section 6.6.

8. False. For example, with $n = 1$ the Trapezoidal Rule is much more accurate than the Midpoint Rule for the function in the diagram.



9. (a) **True.** See the end of Section 6.4.

(b) **False.** Examples include the functions $f(x) = e^{x^2}$, $g(x) = \sin(x^2)$, and $h(x) = \dfrac{\sin x}{x}$.

10. True. If f is continuous on $[0, \infty)$, then $\int_0^1 f(x)\,dx$ is finite. Since $\int_1^\infty f(x)\,dx$ is finite, so is $\int_0^\infty f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^\infty f(x)\,dx$.

11. False. If $f(x) = 1/x$, then f is continuous and decreasing on $[1, \infty)$ with $\lim_{x\to\infty} f(x) = 0$, but $\int_1^\infty f(x)\,dx$ is divergent.

12. True.

$$\int_a^\infty [f(x)+g(x)]\,dx = \lim_{t\to\infty} \int_a^t [f(x)+g(x)]\,dx = \lim_{t\to\infty} \left(\int_a^t f(x)\,dx + \int_a^t g(x)\,dx\right)$$

$$= \lim_{t\to\infty} \int_a^t f(x)\,dx + \lim_{t\to\infty} \int_a^t g(x)\,dx \quad \left[\begin{array}{l}\text{since both limits}\\ \text{in the sum exist}\end{array}\right]$$

$$= \int_a^\infty f(x)\,dx + \int_a^\infty g(x)\,dx$$

Since the two integrals are finite, so is their sum.

13. False. Take $f(x) = 1$ for all x and $g(x) = -1$ for all x. Then $\int_a^\infty f(x)\,dx = \infty$ [divergent] and $\int_a^\infty g(x)\,dx = -\infty$ [divergent], but $\int_a^\infty [f(x) + g(x)]\,dx = 0$ [convergent].

14. False. $\int_0^\infty f(x)\,dx$ could converge or diverge. For example, if $g(x) = 1$, then $\int_0^\infty f(x)\,dx$ diverges if $f(x) = 1$ and converges if $f(x) = 0$.