1. False. See Note 2 on page 441.

2. False. The series $\sum_{n=1}^{\infty} n^{-\sin 1} = \sum_{n=1}^{\infty} \frac{1}{n^{\sin 1}}$ is a p-series with $p = \sin 1 \approx 0.84 < 1$, so the series diverges.

3. True. If $\lim_{n\to\infty} a_n = L$, then as $n \to \infty$, $2n+1 \to \infty$, so $a_{2n+1} \to L$.

4. True by Theorem 8.5.3.

   Or: Use the Comparison Test to show that $\sum c_n(-2)^n$ converges absolutely.

5. False. For example, take $c_n = (-1)^n/(n6^n)$.

6. True by Theorem 8.5.3.

7. False, since $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{1}{(n+1)^3} \cdot \frac{n^3}{1}\right| = \lim_{n\to\infty} \left|\frac{n^3}{(n+1)^3} \cdot \frac{1/n^3}{1/n^3}\right| = \lim_{n\to\infty} \frac{1}{(1+1/n)^3} = 1.$ x

8. True, since $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{1}{(n+1)!} \cdot \frac{n!}{1}\right| = \lim_{n\to\infty} \frac{1}{n+1} = 0 < 1.$

9. False. See the note after Example 4 in Section 8.3.

10. True, since $\frac{1}{e} = e^{-1}$ and $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, so $e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$.

11. True. See (8) in Section 8.1.

12. True, because if $\sum |a_n|$ is convergent, then so is $\sum a_n$ by Theorem 8.4.1.

13. True. By Theorem 8.7.5 the coefficient of x^3 is $\frac{f'''(0)}{3!} = \frac{1}{3} \quad\Rightarrow\quad f'''(0) = 2$.

    Or: Use Theorem 8.6.2 to differentiate f three times.

14. False. Let $a_n = n$ and $b_n = -n$. Then $\{a_n\}$ and $\{b_n\}$ are divergent, but $a_n + b_n = 0$, so $\{a_n + b_n\}$ is convergent.

15. False. For example, let $a_n = b_n = (-1)^n$. Then $\{a_n\}$ and $\{b_n\}$ are divergent, but $a_n b_n = 1$, so $\{a_n b_n\}$ is convergent.

16. True by the Monotonic Sequence Theorem, since $\{a_n\}$ is decreasing and $0 < a_n \le a_1$ for all $n \quad\Rightarrow\quad \{a_n\}$ is bounded.

17. True by Theorem 8.4.1. [$\sum (-1)^n a_n$ is absolutely convergent and hence convergent.]

18. True. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1 \quad\Rightarrow\quad \sum a_n$ converges (Ratio Test) $\quad\Rightarrow\quad \lim_{n\to\infty} a_n = 0$ [Theorem 8.2.6].

19. True. $0.99999\ldots = 0.9 + 0.9(0.1)^1 + 0.9(0.1)^2 + 0.9(0.1)^3 + \cdots = \sum_{n=1}^{\infty}(0.9)(0.1)^{n-1} = \dfrac{0.9}{1-0.1} = 1$ by the formula for the sum of a geometric series $[S = a_1/(1-r)]$ with ratio r satisfying $|r| < 1$.

20. True. Since $\lim_{n\to\infty} a_n = 2$, we know that $\lim_{n\to\infty} a_{n+3} = 2$. Thus, $\lim_{n\to\infty}(a_{n+3} - a_n) = \lim_{n\to\infty} a_{n+3} - \lim_{n\to\infty} a_n = 2 - 2 = 0$.

21. True. A finite number of terms doesn't affect convergence or divergence of a series.

22. False. Let $a_n = (0.1)^n$ and $b_n = (0.2)^n$. Then $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}(0.1)^n = \dfrac{0.1}{1-0.1} = \dfrac{1}{9} = A$,
$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty}(0.2)^n = \dfrac{0.2}{1-0.2} = \dfrac{1}{4} = B$, and $\sum_{n=1}^{\infty} a_n b_n = \sum_{n=1}^{\infty}(0.02)^n = \dfrac{0.02}{1-0.02} = \dfrac{1}{49}$, but
$AB = \frac{1}{9}\cdot\frac{1}{4} = \frac{1}{36}$.