

1. False. Consider the curve defined by $x = f(t) = (t - 1)^3$ and $y = g(t) = (t - 1)^2$. Then $g'(t) = 2(t - 1)$, so $g'(1) = 0$, but its graph has a *vertical* tangent when $t = 1$. *Note:* The statement is true if $f'(1) \neq 0$ when $g'(1) = 0$.

2. False. If $x = f(t)$ and $y = g(t)$ are twice differentiable, then $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$.

3. False. For example, if $f(t) = \cos t$ and $g(t) = \sin t$ for $0 \leq t \leq 4\pi$, then the curve is a circle of radius 1, hence its length is 2π , but $\int_0^{4\pi} \sqrt{[f'(t)]^2 + [g'(t)]^2} dt = \int_0^{4\pi} \sqrt{(-\sin t)^2 + (\cos t)^2} dt = \int_0^{4\pi} 1 dt = 4\pi$, since as t increases from 0 to 4π , the circle is traversed twice.

4. False. If $(r, \theta) = (1, \pi)$, then $(x, y) = (-1, 0)$, so $\tan^{-1}(y/x) = \tan^{-1} 0 = 0 \neq \theta$. The statement is true for points in quadrants I and IV.

5. True. The curve $r = 1 - \sin 2\theta$ is unchanged if we rotate it through 180° about O because $1 - \sin 2(\theta + \pi) = 1 - \sin(2\theta + 2\pi) = 1 - \sin 2\theta$. So it's unchanged if we replace r by $-r$. In other words, it's the same curve as $r = -(1 - \sin 2\theta) = \sin 2\theta - 1$.

6. True. The polar equation $r = 2$, the Cartesian equation $x^2 + y^2 = 4$, and the parametric equations $x = 2 \sin 3t$, $y = 2 \cos 3t$ [$0 \leq t \leq 2\pi$] all describe the circle of radius 2 centered at the origin.

7. False. The first pair of equations gives the portion of the parabola $y = x^2$ with $x \geq 0$, whereas the second pair of equations traces out the whole parabola $y = x^2$.

8. True. Consider a hyperbola with focus at the origin, oriented so that its polar equation is $r = \frac{ed}{1 + e \cos \theta}$, where $e > 1$.

The directrix is $x = d$, but along the hyperbola we have $x = r \cos \theta = \frac{ed \cos \theta}{1 + e \cos \theta} = d \left(\frac{e \cos \theta}{1 + e \cos \theta} \right) \neq d$.