Math 192r, Problem Set #8
(due 10/16/01)

1. Define the diagonal of a two-variable generating function

$$F(x,y) = \sum_{m,n} a_{m,n} x^m y^n$$

as the generating function

$$D(t) = \sum_{n} a_{n,n} t^n.$$

It is a theorem (which we will not have time to prove) that the diagonal of any two-variable rational generating function is an algebraic generating function. Verify this claim in the particular case $F(x,y) = 1/(1-x-y) = \sum_{m,n} \frac{(m+n)!}{m!n!} x^m y^n$ by expressing the diagonal $D(t)$ as an algebraic function. Give as good a justification of your formula as you can.

2. Call a sequence of $+1$'s 0's, and -1's *favorable* if every partial sum is non-negative and the total sum is 0. Let $f(n)$ be the number of favorable sequences of length n. Express the generating function $\sum_n f(n)x^n$ as an algebraic function of x.