[Get 75% of students to turn on cameras]

Reminder: The password for assignment #1 is in the syllabus. (DON'T blurt it, but who's found it?)
Don't ask others; if they ask you, don't tell them.

Chat storm: What's the difference between 0 and $\emptyset$?

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Section 1.1: Set Notation and Relations

- Know basic terminology: set, element, finite set, cardinality, subset, proper subset, set equality
- If a set is given in set-builder notation, write down the elements of the set.
- Express a given set in set-builder notation.
- Know the standard symbols for sets: $\mathbb{P}$, $\mathbb{Z}$, $\mathbb{R}$

What is the set $\mathbb{P}$?

What is the set $\mathbb{N}$?

What is the set $\mathbb{Z}$?

What is the set $\mathbb{Q}$?

What is the set $\mathbb{R}$?

What is the set $\mathbb{C}$?

$x \in S$ means x is an element of the set S. (This is defined in the first paragraph of subsection 1.1.1; if you missed it, then you're skimming too quickly.)

$\emptyset$ (also written { }) is the set with no elements.

{1} is a set with one element, namely the number 1; it is different from 1, which is a number, not a set.

Also, {0}, $\emptyset$, and 0 are all different: the first is a set with one element, the second is a set with no elements, and the third is not a set at all!

Two sets *S* and *T* are equal if every element of *S* is equal to some element of *T* and every element of *T* is equal to some element of *S*.

Compare the sets $\{1,2\}$ and $\{2,1\}$. Are they the same or different? (30 seconds)

..?..

Answer: Same. Order doesn't matter!

Compare the sets $\{1,2\}$ and $\{1,2,2\}$. Are they the same or different? (30 seconds)

..?..

Answer: Same. Repetition doesn't matter!

Sometimes we'll want not just sets of numbers but sets of sets of numbers: for example, when we study partitions (section 2.3). In that context, {{1,2},{3,4}} is NOT the same as {{1,3},{2,4}}.

Suppose *A* and *B* are sets (whose elements may themselves be numbers, or sets, or sets-of-sets, etc.).

To test whether *A* is equal to *B* (as sets), just check that every element of *A* is an element of *B* and vice versa. Putting it differently, to test whether *A* is equal to *B*, just check that every element of *A* is equal to some element of *B* and vice versa.

Note that this is a recursive criterion!

To hammer home this criterion, let's consider unnatural but nonetheless instructive examples.

Compare the sets {{1,1,2},{2,3,3}} and {{3,2},{2,1}}. Are they the same or different? (60 seconds)

..?.

Answer: Same, since {1,1,2} = {2,1} and {2,3,3} = {3,2}.

Compare {{1,2,3},{4,5,6}} and {{1,2},{3,4},{5,6}}. Are they the same or different? (30 seconds)

..?..

Answer: Different. The two elements of {{1,2,3},{4,5,6}} are the set {1,2,3} and the set {4,5,6}, neither of which is equal to the set {1,2} or the set {3,4} or the set {5,6}.

$S \subseteq T$ (pronounced "S is a *subset* of T") means every element of S is equal to some element of T.

Note: If $S \subseteq T$ and $T \subseteq S$ then $S = T$. Conversely, if $S = T$ then $S \subseteq T$ and $T \subseteq S$.

Is {1,2,3,4,5,6} a subset of {{1,2,3,4,5,6}}? (30 seconds)

..?..

Answer: No, but {1,2,3,4,5,6} is an *element* of {{1,2,3,4,5,6}}.

"S is *disjoint* from T" means that S and T have no elements in common; that is, there does not exist an x such that $x \in S$ and $x \in T$.

Distinguish between "$\in$" and "$\subseteq$".

$S \subset T$ (pronounced "S is a proper subset of T") means $S \subseteq T$ but $S \neq T$.

Definition: The cardinality of a set S is the number of different elements it has (for instance, $\{3,4\}$ has cardinality 2, and so does $\{3,3,4\}$); we write the cardinality of S as $|S|$.
Examples: $|\emptyset| = 0$; $|\{0\}| = 1$.

Theorem:
If S and T are finite sets with $S = T$, then $|S| = |T|$.
If S and T are finite sets with $S \subseteq T$, then $|S| \leq |T|$.
If S and T are finite sets with $S \subset T$, then $|S| < |T|$.

Group work (try to set up and share the whiteboard):

- Group work: 1.1.5 (8 minutes)

5. Let $A = \{0, 2, 3\}$, $B = \{2, 3\}$, and $C = \{1, 5, 9\}$. Determine which of the following statements are true. Give reasons for your answers.

(a) $3 \in A$

(b) $\{3\} \in A$

(c) $\{3\} \subseteq A$

(d) $B \subseteq A$

(e) $A \subseteq B$

(f) $\emptyset \subseteq C$

(g) $\emptyset \in A$

(h) $A \subseteq A$

Discuss; how did the whiteboard sharing go?

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Questions on section 1.1?

Section 1.2:

Section 1.2: Basic Set Operations

- Know basic terminology: disjoint sets
- Given sets, perform any combination of the following set operations: union, intersection, complement of A relative to B, complement of A

Exercise 1.2.3 asks you to give examples of sets A, B, $C \subseteq \{1,2,3,\ldots 9\}$ such that various equalities between sets hold. The nice surprise is that they always hold.

Here for instance is how you can show that 1.2.3(c) (the equality $(A \cup B)^c = A^c \cap B^c$, using Venn diagrams:

Section 1.2: Basic Set Operations

- Know basic terminology: disjoint sets
- Given sets, perform any combination of the following set operations: union, intersection, complement of A relative to B, complement of A

Questions on section 1.2?

Section 1.3: Cartesian Products and Power Sets

- Compute the Cartesian product of sets.
- Given a set A, compute A^2, A^3, etc.
- Compute the power set of a set.

Group work: Exercise 1.3.3 (4 minutes)

3. List all two-element sets in $\mathcal{P}(\{a, b, c, d\})$

Recall: S is a subset of T if every element of S is equal to some element of T.

$\wp(T)$ is the set of all subsets of T.

Is $\emptyset$ an element of $\wp(\emptyset)$? (30 seconds)

..?..

Answer: Yes. In fact, it's the ONLY element of $\wp(\emptyset)$.

Is $\emptyset$ a subset of $\wp(\emptyset)$? (30 seconds)

..?..

Answer: Yes. In fact, $\emptyset$ is a subset of T, for EVERY set T.

Section 1.3: Cartesian Products and Power Sets

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- Given a set A, compute A^2, A^3, etc.
- Compute the power set of a set.

Questions on section 1.3?