[Get 75% of students to turn on cameras]

[Have students say a few words about themselves: 10 to 30 seconds each]

Are you able to view the lecture videos?

Section summaries: no photocopying, copying-and-pasting

Questions about definitions.pdf?

Section 1.3: Cartesian Products and Power Sets

- Compute the Cartesian product of sets.
- Given a set A, compute A^2, A^3, etc.
- Compute the power set of a set.

Recall:

S is a subset of *T* if every element of *S* is equal to some element of *T*.

$\wp(T)$ is the set of all subsets of *T*.

E.g., $\wp(\{1,2\}) = \{ \{1,2\}, \{1\}, \{2\}, \{ \} \}$

So *S* is an *element* of $\wp(T)$ if and only if *S* is a *subset* of *T*.

Is $\{1\}$ an element of $\wp(\{1,2\})$?

Yes, because $\{1\}$ is a subset of $\{1,2\}$.

Is 1 an element of $\wp(\{1,2\})$?

No, because 1 is *not* a subset of $\{1,2\}$.

Chat storm: Is $\emptyset$ an element of $\wp(\emptyset)$? (30 seconds)

..?..

Answer: Yes. In fact, it's the ONLY element of $\wp(\emptyset)$.

Chat storm: Is $\emptyset$ a subset of $\wp(\emptyset)$? (30 seconds)

..?..

Answer: Yes. In fact, $\emptyset$ is a subset of T, for EVERY set T.

Section 1.3: Cartesian Products and Power Sets

- Compute the Cartesian product of sets.
- Given a set A, compute A^2, A^3, etc.
- Compute the power set of a set.

Questions on section 1.3?

Section 1.4: Binary Representations of Positive Integers

- Given an integer's binary representation, convert to the decimal representation.
- Use Algorithm 1.4.2 to convert a positive integer to binary.

We have two algorithms for converting from decimal to binary: one constructs the bits in the binary expansion from left to right, and the other constructs them from right to left.

How do we find the bits from left to right?

..?..

Repeatedly subtract off the largest power of 2 you can.

[Do the example n=26, with input from the class.]

Example: 26 is greater than or equal to 1, 2, 4, 8, and 16, but not 32, so the first power of 2 we subtract is 16.

26 – 16 = 10.

Can we subtract 8 (the next smallest power of 2)? Yes!

10 – 8 = 2.

Can we subtract 4 (the next smallest power of 2)? No.

Can we subtract 2 (the next smallest power of 2)? Yes!

2 – 2 = 0.

We've reached 0, so we can stop.

We have 26 – 16 – 8 – 2 = 0.

So 26 = 16 + 8 + 2 = **1**×16 + **1**×8 + **0**×4 + **1**×2 + **0**×1, so 26 in binary is **11010**.

How do we find the bits from right to left?

..?..

Repeatedly divide by 2 and record remainders.

[Do the example *n*=26, with input from the class.]

26÷2 = 13 r **0**

13÷2 = 6 r **1**

6÷2 = 3 r **0**

3÷2 = 1 r **1**

1÷2 = 0 r **1**

So 26 in binary is **11010**.

[Discuss reverse procedures for converting binary to decimal.]

Breakout rooms (tell me if I've got this right):
The person who sets up the whiteboard selects "Share content" and "Whiteboard".
The other people select "View options" and "Annotate".

Group work (5 minutes): Convert 45 to binary. (If time permits, do it both ways.)

Group work (5 minutes): Convert 1110010 to decimal. (If time permits, do it both ways.)

Moral: After you've solved a problem, to make sure you're right, solve it a different way and see if you get the same answer, or work the problem backwards from the answer to the question, or both. This is a much better way to catch your own mistakes than by just taking the exact same approach a second time!

If you give a calculator a problem, it converts your decimal inputs into binary, does binary arithmetic, and converts the binary answer back to decimal.

Examples:

```
  1 1           1 1
+ 1 1         × 1 1
------      ---------
1 1 0           1 1
              1 1
            ---------
             1 0 0 1
```

Section 1.4: Binary Representations of Positive Integers

- Given an integer's binary representation, convert to the decimal representation.
- Use Algorithm 1.4.2 to convert a positive integer to binary.

Questions on section 1.4?

Section 1.5: Summation Notation and Generalizations

- Do computations involving summation notation and generalizations to products and set operations.

Questions on section 1.5?

Group work (5 minutes): Calculate

(a) $\sum_{i=1}^{3}(2+3i)$

Group work (6 minutes):

9. The symbol Π is used for the product of numbers in the same way that Σ is used for sums. For example, $\prod_{i=1}^{5} x_i = x_1 x_2 x_3 x_4 x_5$. Evaluate the following:

(a) $\prod_{i=1}^{3} i^2$

(b) $\prod_{i=1}^{3}(2i+1)$

Note that the use of generalized summation notation implicitly assumes that the operation in question is associative; we wouldn't write

$$*_{n=1}^{3} x_n$$

to mean $x_1 * x_2 * x_3$ if $(x_1 * x_2) * x_3$ and $x_1 * (x_2 * x_3)$ were different.

Group work (8 minutes): Why is $\oplus$ associative? Try to find a simple description of

$$A_1 \oplus A_2 \oplus \cdots \oplus A_n = \bigoplus_{i=1}^{n} A_i$$

Do the same for the XOR (exclusive OR) of n bits.

The XOR of bits $a_1, a_2, \ldots, a_n$ is 1 if an odd number of the a_i's equal 1 and is 0 otherwise; likewise, the $\oplus$-sum of a bunch of sets $A_1, A_2, \ldots, A_n$ is the set of things that belong to an odd number of the A_i's.

Questions on chapter 1?