[Get 75% of students to turn on cameras]

Section 2.3: Partitions of Sets and the Law of Addition

- Know basic terminology: partition

Questions on section 2.3?

What is a partition of a set?

Which of the following collections are partitions of the set {1,2,3,4}?

Polls (30 seconds each):

Is {{1,3},{2,4}} a partition of {1,2,3,4}?

..?..

Yes.

Is {{1,2,3},{2,3,4}} a partition of {1,2,3,4}?

..?..

No (the sets are not disjoint).

Is {{1,2},{3},{4},{}} a partition of {1,2,3,4}?

..?..

No (one of the sets is empty)

Is {{1,3},{2}} a partition of {1,2,3,4}?

..?..

No (the union of the sets is not {1,2,3,4}).

Is {{1,2},{3,4,5}} a partition of {1,2,3,4}?

..?..

No (the union of the sets is not {1,2,3,4})

Is {{1},{2},{3},{4}} a partition of {1,2,3,4}?

..?..

Yes

Is $\{\{1,2,3,4\}\}$ a partition of $\{1,2,3,4\}$?

..?..

Yes

Is $\{1,2,3,4\}$ a partition of $\{1,2,3,4\}$?

..?..

No (this is a collection of numbers, not a collection of subsets of $\{1,2,3,4\}$).

Group work: 2.3.8(a)

A survey of 300 people found that 60 owned an iPhone, 75 owned a Blackberry, and 30 owned an Android phone. Furthermore, 40 owned both an iPhone and a Blackberry, 12 owned both an iPhone and an Android phone, and 8 owned a Blackberry and an Android phone. Finally, 3 owned all three phones.

(a) How many people surveyed owned none of the three phones?

..?..

[Show the Venn diagram method and the algebraic method.]

Other questions on section 2.3?

Section 2.4: Combinations and the Binomial Theorem

- Use the binomial coefficient to compute the number of k-element subsets of an n-element set.
- Use the binomial coefficient to compute of bit strings of length n with exactly k ones. Be able to use the same idea on analogous problems (e.g. counting the number of sequences of n coin flips with exactly k heads).
- Use the binomial coefficient and rule of products in counting problems. Examples:
  - Counting the number of strings with exactly k_1 of one letter, exactly k_2 of a second letter, etc. (e.g. How many different strings can be formed by rearranging the letters in the word CALCULUS?)
  - Counting the number of different groups with m_1 elements from one set and m_2 elements from a second set. (e.g. If a class contains four seniors and six juniors, how many groups of five students can be formed which contain three seniors and two juniors?)
- Use the binomial theorem to compute the expansion of $(x+y)^n$ and related expressions (e.g. $(4a-3b)^5$). Be able to extract a specific term from this expansion.

Questions on section 2.4?

Questions about Example 2.4.5?

Example 2.4.5 Flipping Coins. Assume an evenly balanced coin is tossed five times. In how many ways can three heads be obtained? This is a combination problem, because the order in which the heads appear does not matter. We can think of this as a situation involving sets by considering the set of flips of the coin, 1 through 5, in which heads comes up. The number of ways to get three heads is $\binom{5}{3} = \frac{5 \cdot 4}{2 \cdot 1} = 10$. □

Questions about Example 2.4.6?

Example 2.4.6 Counting five ordered flips two ways. We determine the total number of ordered ways a fair coin can land if tossed five consecutive times. The five tosses can produce any one of the following mutually exclusive, disjoint events: 5 heads, 4 heads, 3 heads, 2 heads, 1 head, or 0 heads. For example, by the previous example, there are $\binom{5}{3} = 10$ sequences in which three heads appear. Counting the other possibilities in the same way, by the law of addition we have:

$$\binom{5}{5} + \binom{5}{4} + \binom{5}{3} + \binom{5}{2} + \binom{5}{1} + \binom{5}{0} = 1 + 5 + 10 + 10 + 5 + 1 = 32$$

ways to observe the five flips.

Of course, we could also have applied the extended rule of products, and since there are two possible outcomes for each of the five tosses, we have $2^5 = 32$ ways. □

Questions about Example 2.4.7?

Example 2.4.7 A Committee of Five. A committee usually starts as an unstructured set of people selected from a larger membership. Therefore, a committee can be thought of as a combination. If a club of 25 members has a five-member social committee, there are $\binom{25}{5} = \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21}{5!} = 53130$ different possible social committees. If any structure or restriction is placed on the way the social committee is to be selected, the number of possible committees will probably change. For example, if the club has a rule that the treasurer must be on the social committee, then the number of possibilities is reduced to $\binom{24}{4} = \frac{24 \cdot 23 \cdot 22 \cdot 21}{4!} = 10626$.

If we further require that a chairperson other than the treasurer be selected for the social committee, we have $\binom{24}{4} \cdot 4 = 42504$ different possible social committees. The choice of the four non-treasurers accounts for the factor $\binom{24}{4}$ while the need to choose a chairperson accounts for the 4. □

Group work: Exercise 2.4.5 (10 minutes)

The image below shows a 6 by 6 grid and an example of a **lattice path** that could be taken from $(0, 0)$ to $(6, 6)$, which is a path taken by traveling along grid lines going only to the right and up. How many different lattice paths are there of this type? Generalize to the case of lattice paths from $(0, 0)$ to (m, n) for any nonnegative integers m and n.

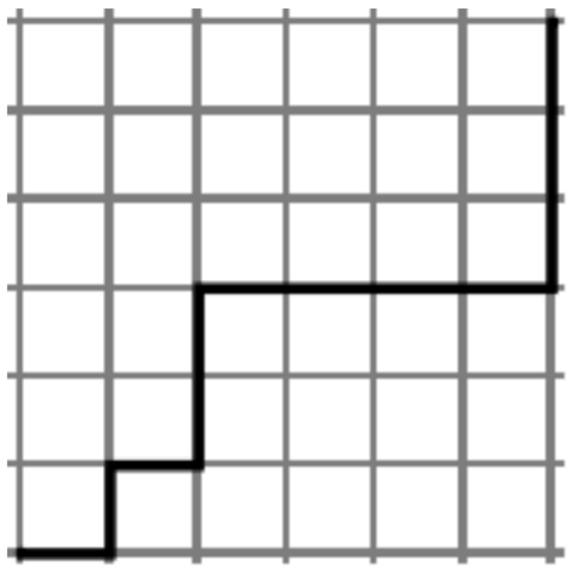

Figure 2.4.12 A lattice path

Hint. Think of each path as a sequence of instructions to go right (R) and up (U).

My trick for computing $C(n,k)$: First write the numerator, which is the product of the k integers that start from n and count down by 1. Then write the denominator, which is the product of the first k positive integers. Then cross out common factors between numerator and denominator. Finally, do some arithmetic to simplify. If you don't get a positive integer, you made a mistake!

Do 12-choose-6 this way.

Group work: Exercise 2.4.6 (10 minutes)

How many of the lattice paths from $(0, 0)$ to $(6, 6)$ pass through $(3, 3)$ as the one in Figure 12 does?

Other questions on section 2.4?