[Get 75% of students to turn on cameras]

Section 3.3: Equivalence and Implication

- Know basic terminology: tautology, contradiction, equivalence, implication.
- Know the notation for tautology (1) and contradiction (0).
- Determine if a proposition is a tautology, contradiction, or neither. Justify the answer using a truth table.
- Given two propositions, determine if one proposition implies the other. Justify the answer using a truth table.
- Given two propositions, determine if the propositions are equivalent. Justify the answer using a truth table.

Questions on 3.3?

Define and discuss terminology

[Discuss “difference” between $\rightarrow$ and $\Rightarrow$.]

Group work: 3.3.2(a) (6 minutes)

[**reset default to 30 seconds before opening rooms; tell students how to Annotate**]

Construct the truth table for $x = (p \wedge \neg q) \vee (r \wedge p)$.

p	q	r	$\neg q$	$p \wedge \neg q$	$r \wedge p$	$(p \wedge \neg q) \vee (r \wedge p)$
0	0	0	1	0	0	0
0	0	1	1	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	0	0	0	0
1	1	1	0	0	1	1

Group work (8 min.): Prove $p \wedge q \to \neg r \iff p \wedge r \to \neg q$ with a truth table.

..?..

p	q	r	$\neg q$	$\neg r$	$p \wedge q$	$p \wedge r$	$p \wedge q \to \neg r$	$p \wedge r \to \neg q$
0	0	0	1	1	0	0	1	1
0	0	1	1	0	0	0	1	1
0	1	0	0	1	0	0	1	1
0	1	1	0	0	0	0	1	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	1	1	1
1	1	0	0	1	1	0	1	1
1	1	1	0	0	1	1	0	0

If we had an extra column for $p \wedge q \to \neg r \iff p \wedge r \to \neg q$ it'd be all 1's, but we don't need to include it; it's enough to check that the last two columns in the table above are identical.

(Later today we'll see a proof that uses the laws of logic.)

Group work: 3.3.6 (10 minutes)

Find a proposition that is equivalent to $p \vee q$ and uses only conjunction and negation.

Answer:

"$p \vee q$" is equivalent to "$\neg (\neg p \wedge \neg q)$"

[Explain the brute force solution. It's kind of like the broadcast algorithm. There are 16 "nodes" to keep track of.]

Other questions on 3.3?

Section 3.4: Laws of Logic

- Understand the laws of logic. Justify why they work using truth tables. (Memorizing the laws is not required.)

Questions on 3.4?

What are DeMorgan's Laws:

..?..

$\neg(p \vee q) \Leftrightarrow (\neg p) \wedge (\neg q)$

And?

..?..

$\neg(p \wedge q) \Leftrightarrow (\neg p) \vee (\neg q)$

Let's use DeMorgan's Laws and the law

$p \to q \Leftrightarrow \neg p \lor q$

to redo the second group work result.

$$
\begin{aligned}
p \land q \to \neg r \quad &\Leftrightarrow \neg (p \land q) \lor \neg r \\
&\Leftrightarrow \neg p \lor \neg q \lor \neg r \\
&\Leftrightarrow \neg p \lor \neg r \lor \neg q \\
&\Leftrightarrow \neg (p \land r) \lor \neg q \\
&\Leftrightarrow p \land r \to \neg q
\end{aligned}
$$

Duality:

The duality principle says that if you have a tautology of the form "$P \Rightarrow Q$", where P and Q are compound propositions of some kind, then "dual-of-$P \Leftarrow$ dual-of-Q" is also a tautology. Also, if "$P \Leftrightarrow Q$" is a tautology, then so is "dual-of-$P \Leftrightarrow$ dual-of-Q".

Group work: 3.4.4(a) (4 minutes)

Write the dual of the following statements:

(a) $(p \wedge q) \Rightarrow p$

$p \vee q \Leftarrow p$ or $p \Rightarrow p \vee q$

The dual of $0 \Rightarrow p$ is $1 \Leftarrow p$.

The dual of "$p \Rightarrow (p \vee q)$" is "$p \Leftarrow (p \wedge q)$", which is equivalent to "$(p \wedge q) \Rightarrow p$".

This shouldn't surprise us, since we learned from 3.4.4(a) that the dual of "$(p \wedge q) \Rightarrow p$" is "$p \Rightarrow (p \vee q)$"!

The dual of "$(p \wedge 1) \Leftrightarrow p$" is "$(p \vee 0) \Leftrightarrow p$".

Note: The duality principle does NOT apply to tautologies that aren't of the form "$P \Rightarrow Q$" or "$P \Leftarrow Q$" or "$P \Leftrightarrow Q$".

Example 1: $p \vee 1$ is a tautology, but $p \wedge 0$ is not.

Example 2: 1 is a tautology, but 0 is not.

Also note: don't confuse negation with dualization:

the *dual* of "$p \vee q$" is "$p \wedge q$", but

the *negation* of "$p \vee q$" is "$\neg p \wedge \neg q$".

Group work: 3.4.4(b) (4 minutes)

4. Write the dual of the following statements:

(b) $(p \vee q) \wedge \neg q \Rightarrow p$

$(p \wedge q) \vee \neg q \Leftarrow p$, or $p \Rightarrow (p \wedge q) \vee \neg q$