**[Get 75% of students to turn on cameras]**

How is everybody doing?

Are the tutors helpful?

**Section 4.1: Methods of Proof for Sets**

- Write proofs using the definitions to show that one set is a subset of another set, or to show that two sets are equal.

Questions about 4.1?

Discuss proofs using membership tables:

**Table 4.1.6 Membership table to prove the distributive law of intersection over union**

| $A$ | $B$ | $C$ | $B \cup C$ | $A \cap B$ | $A \cap C$ | $A \cap (B \cup C)$ | $(A \cap B) \cup (A \cap C)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Questions about the proof of Theorems 4.1.7 and 4.1.8?

Theorem 4.1.8: $A \times (B \cap C) = A \times B \cap A \times C$.

Discuss proof:

(a) $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$.
Let $(x, y) \in A \times (B \cap C)$.

$(x, y) \in A \times (B \cap C) \Rightarrow x \in A$ and $y \in (B \cap C)$

Why?

$\Rightarrow x \in A$ and ($y \in B$ and $y \in C$)

Why?

$\Rightarrow$ ($x \in A$ and $y \in B$) and ($x \in A$ and $y \in C$)

Why?

$\Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$

Why?

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

Why?

(b) $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$.
Let $(x, y) \in (A \times B) \cap (A \times C)$.

$(x, y) \in (A \times B) \cap (A \times C) \Rightarrow (x, y) \in A \times B$ and $(x, y) \in A \times C$

Why?

$\Rightarrow$ ($x \in A$ and $y \in B$) and ($x \in A$ and $y \in C$)

Why?

$\Rightarrow x \in A$ and ($y \in B$ and $y \in C$)

Why?

$\Rightarrow x \in A$ and $y \in (B \cap C)$

Why?

$\Rightarrow (x, y) \in A \times (B \cap C)$

Why?

Discuss two ways of talking about sets: plural language ("all elements of $A$ are in $B$", etc.) and singular language ("each element of $A$ is in $B$").

Group work: 4.1.1(a) (5 minutes)

Let $A$, $B$, and $C$ be sets. Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

..?..

Assume $x \in A$.

Since $x \in A$ and $A \subseteq B,\ x \in B$.

Since $x \in B$ and $B \subseteq C,\ x \in C$.

So every element of $A$ is an element of $C$.

Hence $A \subseteq C$.

Group work: 4.1.1(c) (4 minutes)

Let $A$, $B$, and $C$ be sets. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

..?..

Assume $x \in A$.

Since $A \subseteq B$ and $A \subseteq C$, $x \in B$ and $x \in C$.

Hence $x \in B \cap C$.

So every element of $A$ is an element of $B \cap C$.
So $A \subseteq B \cap C$.

Chat storm:

What is the converse of “If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$”?

..?..

The converse is “If $A \subseteq C$, then $A \subseteq B$ and $B \subseteq C$.”

Chat storm: Is it true or false?

..?..

The converse is false: A counterexample is $A = \{1,2\}$, $B = \{3,4\}$, $C = \{1,2\}$.

Group work: 4.1.2(c) (4 minutes):

Write the converse of "If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$" and prove or disprove it.

..?..

What is the converse of “If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$”?

..?..

The converse is “If $A \subseteq B \cap C$, then $A \subseteq B$ and $A \subseteq C$.”

Is it true or false?

..?..

The converse is true.

Proof: Let $x \in A$.

Since $x \in A$ and $A \subseteq B \cap C$, $x \in B \cap C$.

So $x \in B$ and $x \in C$.

Since every element of $A$ is an element of $B$ and every element of $A$ is an element of $C$, $A \subseteq B$ and $A \subseteq C$.

Other questions on section 4.1?

**Section 4.2: Laws of Set Theory**

- Be familiar with the laws of set theory.
- Be able to use the laws of set theory to prove new equalities (e.g. problems 3-4).

Questions on section 4.2?

Group work: 4.2.4(b) (8 minutes)

Use previously proven theorems to prove

$$A \cap (B \cap (A \cap B)^c) = \varnothing$$

..?..

My way:

$A \cap (B \cap (A \cap B)^c)$

$= A \cap (B \cap (A^c \cup B^c))$ (De Morgan)

$= A \cap (B \cap A^c \cup B \cap B^c)$ (distributive law)

$= A \cap (B \cap A^c \cup \varnothing)$ (complement law)

$= A \cap (B \cap A^c)$ (identity law)

$= A \cap (A^c \cap B)$ (commutative law)

$= (A \cap A^c) \cap B$ (associative law)

$= \varnothing \cap B$ (complement law)

$= \varnothing$ (null law)

A better way:

..?..

$A \cap (B \cap (A \cap B)^c)$

$= (A \cap B) \cap (A \cap B)^c$ (associative law)

$= \varnothing$ (complement law applied to the set $A \cap B$)

Other questions on section 4.2?