[Get 75% of students to turn on cameras]

Section 4.3: Minsets

- Given a finite list of subsets of a universe, list all minsets. Know that the minsets form a partition (Theorem 4.3.8).

A minset corresponds to a region in a Venn diagram in which the circles correspond to the sets $B_1, B_2, \ldots$

Example: Say B_1 is the set of people who like cats and B_2 is the set of people who like dogs. Then there are $2 \times 2 = 4$ minsets:

the set of people who like both cats and dogs,

the set of people who like cats but not dogs,

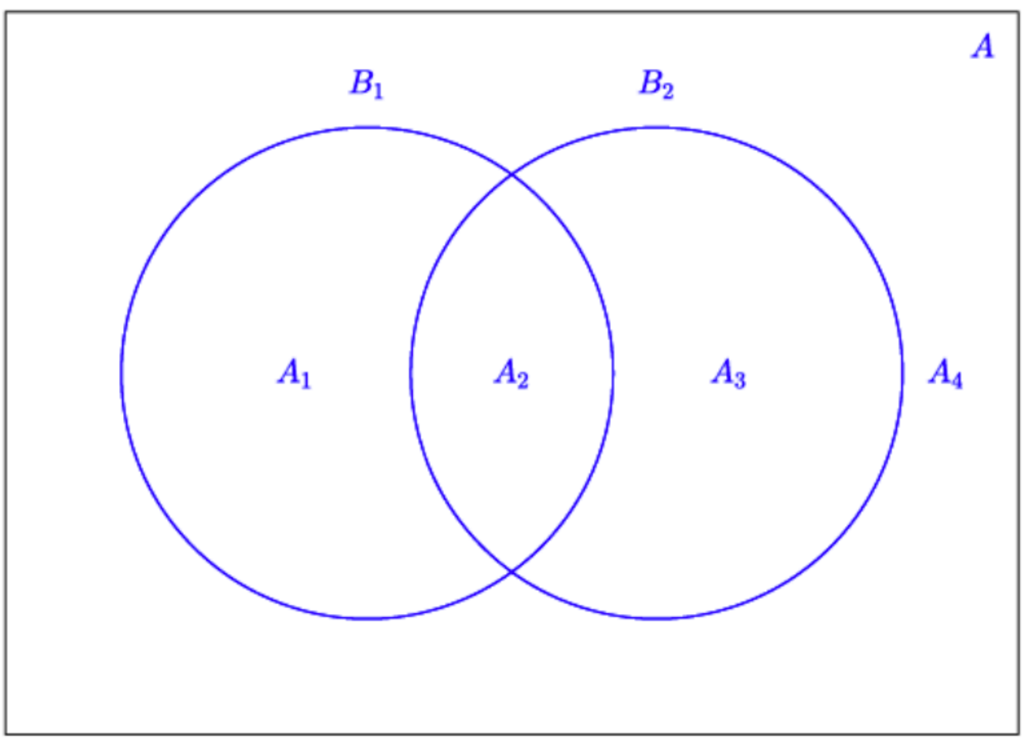
the set of people who don't like cats but do like dogs, and

the set of people who like neither cats nor dogs.

If we have sets B_1, B_2, and B_3, then there are $2 \times 2 \times 2 = 8$ minsets (some of which could be empty).
If we have n sets $B_1, \ldots, B_n$, then there are 2^n minsets.

Figure 4.3.1 may be helpful.

Let's draw a modified version of Figure 4.3.1 pertaining to Example 4.3.5, in which actual elements of U are drawn in the appropriate regions of the Venn diagram.



$A = \{1,2,3,4,5,6\}$, $B_1 = \{1,3,5\}$, $B_2 = \{1,2,3\}$

In the case where there are n sets $B_1, B_2, \ldots, B_n$, the minsets (some of which may be empty!) correspond to intersections $D_1 \cap D_2 \cap \ldots \cap D_n$, where each D_k is either B_k or the complement of B_k. We can list the 2^n minsets systematically, by associating each minset with a string of n bits (where a 0 in the kth position means $D_k = (B_k)^c$ and a 1 there means $D_k = B_k$), and we may in turn associate that string of n bits with an integer between 0 and $2^n - 1$ (via binary notation). E.g., when n=2, we have

0: M_{00}: $(B_1)^c \cap (B_2)^c$ (A_4)

1: M_{01}: $(B_1)^c \cap (B_2)$ (A_3)

2: M_{10}: $(B_1) \cap (B_2)^c$ (A_1)

3: M_{11}: $(B_1) \cap (B_2)$ (A_2)

When we write a set as a union of minsets, we say we are writing it in canonical form (aka minset normal form). For instance, in the case n=2, we can write B_1 as the union of the two minsets

$(B_1) \cap (B_2)^c$ and $(B_1) \cap (B_2)$, and we can write $B_1 \cup B_2$ as the union of the three minsets

$(B_1) \cap (B_2)^c$ and $(B_1) \cap (B_2)$ and $(B_1)^c \cap (B_2)$.

Group work (~6 minutes): What is the minset normal form of ${B_1}^c \cup B_2$? You can write your answer in terms of A_1, A_2, A_3, and A_4, or in terms of M_{00}, M_{01}, M_{10}, and M_{11}, or in terms of $B_1 \cap B_2$, $B_1 \cap (B_2)^c$, $(B_1)^c \cap B_2$, and $(B_1)^c \cap (B_2)^c$; these are all different ways of writing the minsets.

..?..

$({B_1}^c \cap {B_2}^c) \cup ({B_1}^c \cap B_2) \cup (B_1 \cap B_2)$.

If A and B are subsets of the universe U, the canonical form of $A^c \cup B$ is $(A^c \cap B^c) \cup (A^c \cap B) \cup (A \cap B)$.

There is an analogous notion for Boolean formulas. The truth table for the proposition $p \vee q$ tells us that there are three ways for $p \vee q$ to be true: when $(p \wedge q)$ is true, when $(p \wedge \neg q)$ is true, and when $(\neg p \wedge q)$ is true. So the "minterm form" of $p \vee q$ is $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)$.

(Compare and contrast the terms *minset* and *minterm*.)

Just as the minset normal form of $A^c \cup B$ is
$(A^c \cap B^c) \cup (A^c \cap B) \cup (A \cap B)$,
the minterm form of $p \to q$ is
$(\neg p \wedge \neg q) \vee (\neg p \wedge q) \vee (p \wedge q)$.

Questions on section 4.3?

Discussion:

Example 4.3.5 A concrete example of some minsets. Consider the following concrete example. Let $A = \{1, 2, 3, 4, 5, 6\}$ with subsets $B_1 = \{1, 3, 5\}$ and $B_2 = \{1, 2, 3\}$. How can we use set operations applied to and produce a partition of A? [redacted]

How many different *minsets* can be generated from B_1 and B_2?

..?..

4.

How many different *subsets* of A can be generated from B_1 and B_2?

..?..

$2^4 = 16$, since all 4 minsets are nonempty.

More generally, the number of different subsets is equal to 2 to the power of the number of nonempty minsets.

Group work: Exercise 4.3.5 (~6 minutes)

5.

(a) Partition $A = \{0,1,2,3,4,5\}$ with the minsets generated by $B_1 = \{0,2,4\}$ and $B_2 = \{1,5\}$.

(b) How many different subsets of A can you generate from B_1 and B_2?

..?..

(a) {0,2,4}, {1,5}, {3}

(b) $2^3 = 8$

Group work: Exercise 4.3.1 (~6 minutes)

1. Consider the subsets $A = \{1, 7, 8\}$, $B = \{1, 6, 9, 10\}$, and $C = \{1, 9, 10\}$, where $U = \{1, 2, ..., 10\}$.

(a) List the nonempty minsets generated by $A, B,$ and C.

(b) How many elements of the power set of U can be generated by A, B, and C? Compare this number with $\mid \mathcal{P}(U) \mid$. Give an example

of one subset that cannot be generated by A, B, and C.

..?..

(a) {1}, {2,3,4,5}, {6}, {7,8}, {9,10}

(b) $2^5 = 32$

Other questions on section 4.3?

Section 4.4: The Duality Principle

- Given a set equality, state the dual (e.g. problem 1).

Chat storm: 4.4.1(c): (3 min)

State the dual of $(A \cup B^c)^c \cap B = A^c \cap B$ (which could be chatted as “(A union B)c intersect B = Ac intersect B”).

..?..

$(A \cap B^c)^c \cup B = A^c \cup B$

Group work: 4.4.3(b): (3 min)

State the dual of $(\neg(p \wedge (\neg q))) \vee q \Leftrightarrow (\neg p \vee q)$ (which could be chatted as “not(p and (not q)) or q equiv (not-p or q)”.)

..?..

$(\neg(p \vee (\neg q))) \wedge q \Leftrightarrow (\neg p \wedge q)$

Notice that if p and q are propositions over a universe U, and A is the truth set of p and B is the truth set of q, then the two sets $(A \cup B^c)^c \cap B$ and $A^c \cap B$ are the truth sets of the two propositions $(\neg(p \vee (\neg q))) \wedge q$ and $\neg p \wedge q$ while the two sets $(A \cap B^c)^c \cup B$ and $A^c \cup B$ are the truth sets of the two propositions $(\neg(p \wedge (\neg q))) \vee q$ and $\neg p \vee q$. So D&L "lazily" assigned the same problem twice, once as a problem about set theory and once as a problem about propositional logic!

Prove 4.4.1(c) ("$(A \cup B^c)^c \cap B = A^c \cap B$") with a membership table.

A	B	B^c	$A \cup B^c$	$(A \cup B^c)^c$	$(A \cup B^c)^c \cap B$	A^c	$A^c \cap B$
0	0	1	1	0	0	1	0
0	1	0	0	1	1	1	1
1	0	1	1	0	0	0	0
1	1	0	1	0	0	0	0

Other questions on section 4.4? On chapter 4?