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Section 6.4: Matrices of Relations

- Represent a relation using an adjacency matrix.
- Use the adjacency matrix of a relation to find the set of ordered pairs for the relation.
- Do computations using Boolean arithmetic.
- Compute the adjacency matrix of a composition of relations using Theorem 6.4.6. Use this computation to list the ordered pairs in the composition.

Key ideas from section 6.4: adjacency matrix of a relation;

Boolean arithmetic (writing "·" for "∧" and "+" for "∨")

[Draw truth tables for "·", "∧", "+", and "∨"]

When "1+1=1" really means "True ∨ True = True", we refer to "+" as *Boolean addition*.

The Boolean matrix for a relation r from a set A to a set B is the $|A|$-by-$|B|$ matrix whose i,jth entry is 1 or 0 according to whether or not $a_i \, r \, b_j$ is true, where a_i is the ith element of A and b_j is the jth element of B.

[Draw the Boolean matrix for "divides" in D_6.]

When we do Boolean arithmetic on matrices, we use 1+1=1 but other than that we do matrix arithmetic the usual way.

Suppose r and s are relations from a set A to a set B. Then the relation $r \cup s$ (where we view r and s as sets of ordered pairs in $A \times B$) encodes the disjunction "*arb* or *asb*", and its Boolean matrix is the *Boolean* sum (that is, the "1+1=1 sum") of the matrices associated with r and s.

Group work (around 6 minutes): Let $A = B = \{1,2\}$, let $r = \{(1,1),(1,2)\}$, and let $s = \{(1,1),(2,1)\}$, so that $r \cup s = \{(1,1),(1,2),(2,1)\}$. Find the Boolean matrices of r, s, and $r \cup s$, and check that the third matrix is the Boolean sum of the first two.

Key fact: For any relation r from a set A to a set B, and any relation s from a set B to a set C, the Boolean matrix for rs is equal to the product of the (rectangular) Boolean matrix for r and the (rectangular) Boolean matrix for s, as long as we use 1+1=1 arithmetic.

Example: Let r be the relation $\{(1,3),(1,4),(2,4)\}$ from the set $A = \{1,2\}$ to the set $B = \{3,4\}$ and let s be the relation $\{(3,6),(4,6)\}$ from the set $B = \{3,4\}$ to the set $C = \{5,6\}$, so that rs is the composition of the two relations.
Then the Boolean matrix for the relation rs is equal to the product of the Boolean matrix for the relation r and the Boolean matrix for the relation s with 1+1=1 arithmetic.

[Show this pictorially.]

The addition rule 1+1=1 seems less bizarre if we think of 0 and 1 as meaning “0 ways” and “at least 1 way”.

Group work (10 min): 6.4.1

1. Let $A_1 = \{1, 2, 3, 4\}$, $A_2 = \{4, 5, 6\}$, and $A_3 = \{6, 7, 8\}$. Let r_1 be the relation from A_1 into A_2 defined by $r_1 = \{(x, y) \mid y - x = 2\}$, and let r_2 be the relation from A_2 into A_3 defined by $r_2 = \{(x, y) \mid y - x = 1\}$.

(a) Determine the adjacency matrices of r_1 and r_2.

(b) Use the definition of composition to find $r_1 r_2$.

(c) Verify the result in part b by finding the product of the adjacency matrices of r_1 and r_2.

(See textbook for solution.)

Group work (10 minutes): 6.4.5

5. How many different reflexive, symmetric relations are there on a set with three elements?

Hint. Consider the possible matrices.

Other questions on section 6.4?