Definitions

Is zero even? Students often find this problem puzzling. They've only seen the concepts of "even" and "odd" in the context of positive integers, and it seems suspect to extend the concepts beyond the familiar domain of positive integers. But mathematicians have found it helpful to adopt a looser definition, and to declare that "An even integer is an integer that can be written as twice an integer." Thus, the negative integer -6 is regarded as even, because it's twice -3. Likewise, the integer 0 is regarded as even because it's twice 0. From this definition, lots of nice properties follow: for instance, the sum of two even integers is an even integer.

You *could* choose to define the word "even" in a different way (although we should probably use a different word, to avoid confusion!). For instance, you could say "A *steven* integer is an integer that can be written as twice some OTHER integer," and then 0 wouldn't be a steven integer. But that would be a bad definition. What makes it bad? For starters, it would no longer be true that the sum of two steven integers is steven, because 6 and -6 are steven but $6 + (-6)$ is not.

This somewhat silly example illustrates a serious point: in mathematics, we often adopt definitions that make it easier for us to prove useful theorems, or makes it possible for us to get along with fewer theorems.

Another example: Should we define the word "rectangle" to explicitly exclude squares? It's convenient to include squares as a special kind of rectangles, rather than exclude them, because most theorems that are true about rectangles are also true about squares.

Here's a final example: Is 1 prime? Centuries ago, many mathematicians would have said it was. But ultimately mathematics advanced to the point where many of the theorems that are true of primes like 2, 3, 5, etc. were false for the number 1. So it became convenient to choose a different definition of primes that excluded 1. Otherwise, theorem after theorem would have had to say "For all primes bigger than 1" rather than "For all primes".

The common theme here is that definitions aren't God-given; they're made by us humans, and we use the definitions that are convenient for us. It's fine to make a different definition than a standard one, as long as you use a different word (the way I did with "steven") to make it clear that you mean something different from the standard notion.

One last thing: asking "What is the parity of n?" is the same as asking whether n is even or odd.