“Distinct” versus “unique”

We say a mathematical entity is *unique* when there’s nothing else like it. “There exists a unique real number x such that $x^2 = 0$” means there’s only one such number.

We say that two mathematical entities are *distinct* when they are unequal to each other. “There exist distinct real numbers x, y such that $x^2 = y^2$” means you can find real numbers $x \neq y$ satisfying $x^2 = y^2$. More generally, we say that n mathematical entities are distinct when no two of them are equal.

Usually we apply the word “unique” to a single thing, and “distinct” to two or more things. But there are exceptions. For instance, we say “There exist unique x, y such that $\ldots$” to mean that there’s only one ordered pair (x, y) satifying a specified condition.

Here are a few (somewhat artificial) examples to highlight the difference between these two words:

“There exist distinct integers x, y such that $x^2 + y^2 = 1$” is true (just take $x = 0$ and $y = 1$, for instance; there are other solutions but just one will do to prove existence);

“There exist unique integers x, y such that $x^2 + y^2 = 1$” is false (there are four solutions $(x, y) = (0, 1)$, $(0, -1)$, $(1, 0)$, and $(-1, 0)$, though finding just two solutions suffices to prove non-uniqueness);

“There exist distinct integers x, y such that $x^2 + y^2 = 0$” is false; and

“There exist unique integers x, y such that $x^2 + y^2 = 0$” is true.

Moral: “distinct” and “unique” mean very different things.

A symbol we sometimes use to assert “There exists a unique x” is $\exists! x$; e.g., we write $(\exists! x)(\exists! y)\ x^2 + y^2 = 0$.