

1. True. $f_y(a, b) = \lim_{h \rightarrow 0} \frac{f(a, b+h) - f(a, b)}{h}$ from Equation 11.3.3. Let $h = y - b$. As $h \rightarrow 0$, $y \rightarrow b$. Then by substituting,

$$\text{we get } f_y(a, b) = \lim_{y \rightarrow b} \frac{f(a, y) - f(a, b)}{y - b}.$$

2. False. If there were such a function, then $f_{xy} = 2y$ and $f_{yx} = 1$. So $f_{xy} \neq f_{yx}$, which contradicts Clairaut's Theorem.

3. False. $f_{xy} = \frac{\partial^2 f}{\partial y \partial x}$.

4. True. From Equation we get $D_{\mathbf{k}} f(x, y, z) = \nabla f(x, y, z) \cdot \langle 0, 0, 1 \rangle = f_z(x, y, z)$.

5. False. See Example 11.2.3.

6. False. See Exercise 11.4.38(a).

7. True. If f has a local minimum and f is differentiable at (a, b) then by Theorem 11.7.2, $f_x(a, b) = 0$ and $f_y(a, b) = 0$, so

$$\nabla f(a, b) = \langle f_x(a, b), f_y(a, b) \rangle = \langle 0, 0 \rangle = \mathbf{0}.$$

8. False. If f is not continuous at $(2, 5)$, then we can have $\lim_{(x,y) \rightarrow (2,5)} f(x, y) \neq f(2, 5)$. (See Example 11.2.7)

9. False. $\nabla f(x, y) = \langle 0, 1/y \rangle$.

10. True. This is part (c) of the Second Derivatives Test (11.7.3).

11. True. $\nabla f = \langle \cos x, \cos y \rangle$, so $|\nabla f| = \sqrt{\cos^2 x + \cos^2 y}$. But $|\cos \theta| \leq 1$, so $|\nabla f| \leq \sqrt{2}$. Now

$$D_{\mathbf{u}} f(x, y) = \nabla f \cdot \mathbf{u} = |\nabla f| |\mathbf{u}| \cos \theta, \text{ but } \mathbf{u} \text{ is a unit vector, so } |D_{\mathbf{u}} f(x, y)| \leq \sqrt{2} \cdot 1 \cdot 1 = \sqrt{2}.$$

12. False. See Exercise 11.7.29.