Math 305, Problem Set #12
(due **by email** Monday, 12/14/09, 10:30 a.m.)

Abbott, section 7.2, problems 2, 4(bc).

Abbott, section 7.3, problems 1, 2, and 5.

Abbott, section 7.4, problems 1, 4, 6(abc). Hint: For problem 1(a), use Exercise 1.2.5(b).

Extra problem A: Define modified lower and upper integrals $L^*(f) = \sup\{L^*(f,P)\}$ and $U^*(f) = \inf\{U^*(f,P)\}$ based on modified lower and upper sums $L^*(f,P) = \sum_{1 \le k \le n} m_k^*(x_k - x_{k-1})$ and $U^*(f,P) = \sum_{1 \le k \le n} M_k^*(x_k - x_{k-1})$, where $m_k^* = \inf\{f(x) : x \in (x_{k-1}, x_k)\}$ and $M_k^* = \sup\{f(x) : x \in (x_{k-1}, x_k)\}$. Show that for every function f on $[a,b]$, $L^*(f) = L(f)$ and $U^*(f) = U(f)$. (That is, this approach gives the same notion of integrability as the one Abbott describes.)

Note: In your solution to a problem, you may appeal to the results proved on the homework in earlier problem sets or the current problem set (as long as you don't engage in circular reasoning).

Please don't forget to write down **who you worked on the assignment with** (if nobody, then write "I worked alone"), and record **how much time you spent on each problem** (this doesn't need to be exact).