Math 305, Problem Set #2
(due **by email** Friday, 9/25/09)

Abbott, section 1.3, problems 2, 3(a), 8, and 9. (For problem 3(a), add the stipulation that A is non-empty.)

Abbott, section 1.4, problem 5.

Extra problem A: Return to the sequence defined in problem 1.2.10, and find (with proof) the supremum of $\{y_n : n \in \mathbf{N}\}$.

Extra problem B: Give a different proof of Theorem 1.4.1 by taking $x = \inf B$ with $B = \{b_n : n \in \mathbf{N}\}$.

Extra problem C: Show that the set $\{r\sqrt{2} \, : \, r \in \mathbf{Q}\}$ is dense in $\mathbf{R}$.

Extra problem D: For each $n \in \mathbf{N}$, assume we are given rational numbers $a_n \leq b_n$, so that $J_n = \{x \in \mathbf{Q} : a_n \leq x \leq b_n\}$ is non-empty. Assume also that each J_n contains J_{n+1}, so that $J_1 \supseteq J_2 \supseteq J_3 \supseteq \ldots$. Must it be the case that the intersection $\cap_{n=1}^{\infty} J_n$ is non-empty? (Of course you must prove your answer!)

Please don't forget to write down **who you worked on the assignment with** (if nobody, then write "I worked alone"), and record **how much time you spent on each problem** (this doesn't need to be exact) on the time-sheets I'll give out in class on Sept. 10.