Math 305, Problem Set #4
(due **by email** Friday, 10/16/09, 10:30 a.m.)

Abbott, section 2.7, problems 1(a), 4, and 6.

Abbott, section 3.2, problems 1, 3, 7, 11, and 12.

Extra problem A: Imitating Abbott's proof of the uncountability of $\mathbf{R}$ (pp. 25–26), prove the uncountability of the Cantor set C.

Extra problem B: What is the flaw in the following induction proof? "Claim: If $x_1 = 1$ and $x_{n+1} = x_n + 1$ for all $n \geq 1$, then the sequence (x_n) is decreasing; that is, $x_n \geq x_{n+1}$ for all n in $\mathbf{N}$. Proof: Suppose $x_n \geq x_{n+1}$. Then adding 1 to both sides gives

$$x_n + 1 \geq x_{n+1} + 1.$$

But since $x_n + 1 = x_{n+1}$ and $x_{n+1} + 1 = x_{n+2}$, this is equivalent to

$$x_{n+1} \geq x_{n+2}.$$

Since we have shown that, for all n, $x_n \geq x_{n+1}$ implies $x_{n+1} \geq x_{n+2}$, the Principle of Induction tells us that $x_n \geq x_{n+1}$ for all n." Note that it is *not* enough to say that the proof must be wrong because the claim is false; I want you to tell me exactly where the proof goes astray!

Please don't forget to write down **who you worked on the assignment with** (if nobody, then write "I worked alone"), and record **how much time you spent on each problem** (this doesn't need to be exact).