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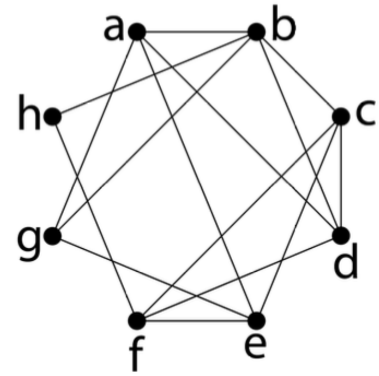
Remember to put on and turn on mic and turn up volume.

The final exam will be on Friday, May 8th in Ball 210 from 6:30pm to 9:30pm.

1. Let G be the given graph.

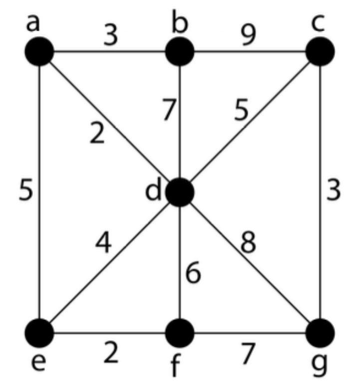


   (a) Find a Hamiltonian circuit in G.

   (b) Use the appropriate theorem to explain why G does not have an Eulerian circuit.

   (c) Remove an edge from G to get a new graph H that does have an Eulerian circuit. Then list (in order) the vertices in an Eulerian circuit in H.

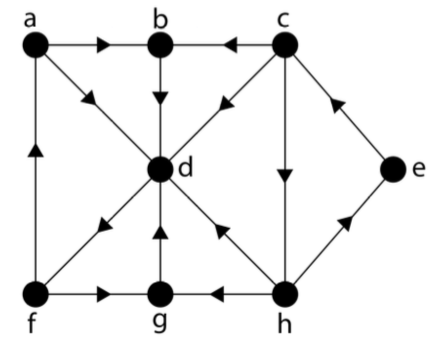
2. For the given weighted graph G:



(a) Find all bridges between $L = \{a, b, d\}$ and $R = \{c, e, f, g\}$. Then find the bridge of minimum weight.

(b) Use Prim's algorithm to find a minimal spanning tree. Start with the vertex a. List the vertices and edges as they're added to the spanning tree. Then find the weight of this tree.

3. For the given directed graph G:



(a) Use the breadth-first algorithm from section 9.3 to find the shortest path from the vertex a to the vertex e (if possible). Fill out the table below:

Vertex k	a	b	c	d	e	f	g	h
V_k.found								
V_k.from								
Depth Set								

(b) Use the table from part (a) to find the shortest path from the vertex a to the vertex g.

(c) Is the graph G connected? strongly connected?

4. Suppose a binary operation $\diamond$ on the set $A = \{x, y, z\}$ is given by the table below:

$\diamond$	x	y	z
x	y	z	x
y	z	y	y
z	x	y	z

Which of the following properties does $\diamond$ have? Explain your answers.

(a) commutative

(b) identity

(c) inverse

5. Let $[G; *]$ be a group, and let $a, b, c \in G$.

(a) Solve the equation $b * x * a^2 = a * c * a$ for $x \in G$. Simplify your answer.

(b) Expand and simplify the following expression: $(b * c * b^{-1})^3$

6. In each part, a group $[G; *]$ and two elements X, Y of G are given. Compute X^4, X^{-3}, and $(X * Y)^{-1}$. Also, state the identity for each group. (*Note:* Each group has a standard binary operation – use this for $*$ in each part.)

(a) Group: $\mathbb{Z}_6 \times \mathbb{Z}_8$
Elements: $X = (3, 5)$, $Y = (2, 7)$

6. In each part, a group $[G; *]$ and two elements X, Y of G are given. Compute X^4, X^{-3}, and $(X * Y)^{-1}$. Also, state the identity for each group. (*Note:* Each group has a standard binary operation – use this for $*$ in each part.)

(b) Group: $\mathbb{R}^* \times \mathbb{R} \times \mathbb{R}$
Elements $X = (3, 1, 4), \quad Y = (7, 2, 5).$

7. For the group in part (a) of problem 6, compute the cyclic subgroup generated by X and the order of X.

(a) Group: $\mathbb{Z}_6 \times \mathbb{Z}_8$
Elements: $X = (3, 5)$, $Y = (2, 7)$

8. For the group in part (b) of problem 6, compute five different elements in the cyclic group generated by X, and compute the order of X.

(b) Group: $\mathbb{R}^* \times \mathbb{R} \times \mathbb{R}$

Elements $X = (3, 1, 4), \quad Y = (7, 2, 5)$.

9. Consider the group $\mathbb{U}_9$ with the standard binary operation of $\times_9$.

    (a) Compute $4 \times_9 5$.

    (b) List all of the elements of the group. Then find the inverse of each element.

    (c) Is $\mathbb{U}_9$ cyclic? Explain why or why not.

10. Let $S = \{1, 2, \ldots, 10\}$, and let $\mathcal{P} = \mathcal{P}(S)$ be the power set of S.

(a) Note that $\cup$ (i.e. set union) is a binary operation on $\mathcal{P}$. Is $\mathcal{P}$ a group under this binary operation? Explain which properties of the definition work and which properties fail. (You can assume the laws of set theory are true.)

(b) Note that set complement is a unary operation on $\mathcal{P}$. More precisely, under this operation, the set A is mapped to the set $A^c = S - A$. Is this unary operation an involution? Explain your answer.

11. Use Theorem 11.5.3 (subgroup conditions) to do the problems below.

(a) Prove that the set $H = \{(2k, 3^n) \mid k, n \in \mathbb{Z}\}$ is a subgroup of $\mathbb{R} \times \mathbb{R}^+$.

(b) Explain why the set $H = \{(2k, 3^n) \mid k, n \in \mathbb{Z}, n \geq 0\}$ is not a subgroup of $\mathbb{R} \times \mathbb{R}^+$.