Use www.uml.edu/attendance app.

Remember to put on and turn on mic and turn up volume.

The final exam will be on Friday, May 8th in Ball 210 from 6:30pm to 9:30pm.

12. Let $[G; *]$ be a group, where $G = \{u, v, x, y, z\}$, and suppose there's an isomorphism $f : \mathbb{Z}_5 \to G$ given by $f(0) = v$, $f(1) = y$, $f(2) = u$, $f(3) = x$, $f(4) = z$. Find the following. Express each answer as u, v, x, y or z.

(a) the identity of G.

(b) the inverse of u

(c) $u * z$

(d) x^3

13. Prove that $f : \mathbb{R}^+ \to \mathbb{R}^+$ given by $f(x) = \sqrt{x}$ is an isomorphism using the definition of *isomorphism.*

14. A pair of groups is given in each part. Explain why the groups aren't isomorphic.

(a) $\mathbb{Z}_2 \times \mathbb{Z}$ and $\mathbb{Z} \times \mathbb{Z}$

(b) $\mathbb{Z}_6 \times \mathbb{Z}_4$ and $\mathbb{Z}_3 \times \mathbb{Z}_4$

15. Let $S = \{2, 4, 5, 12, 20, 30, 60\}$. Consider the poset $(S, |)$. That is, consider the set S equipped with the partial order called "divides".

(a) Draw the Hasse diagram for the poset S.

(b) Find the following, if possible. (If it's not possible, say so.)

i. all upper bounds of 2 and 5

ii. $2 \vee 5$

iii. all lower bounds of 12 and 20

iv. $12 \wedge 20$

v. the least element

vi. the greatest element

16. Let $S = \{2, 4, 5, 12, 20\}$, and let $A = \{2, 4, 20\}$ and $B = \{4, 5, 20\}$. Compute the following for the poset $(\mathcal{P}(S), \subseteq)$:

(a) all upper bounds of A and B.

(b) all lower bounds of A and B.

(c) the join of A and B.

(d) the meet of A and B.

(e) the least element of the poset.

(f) the greatest element of the poset.

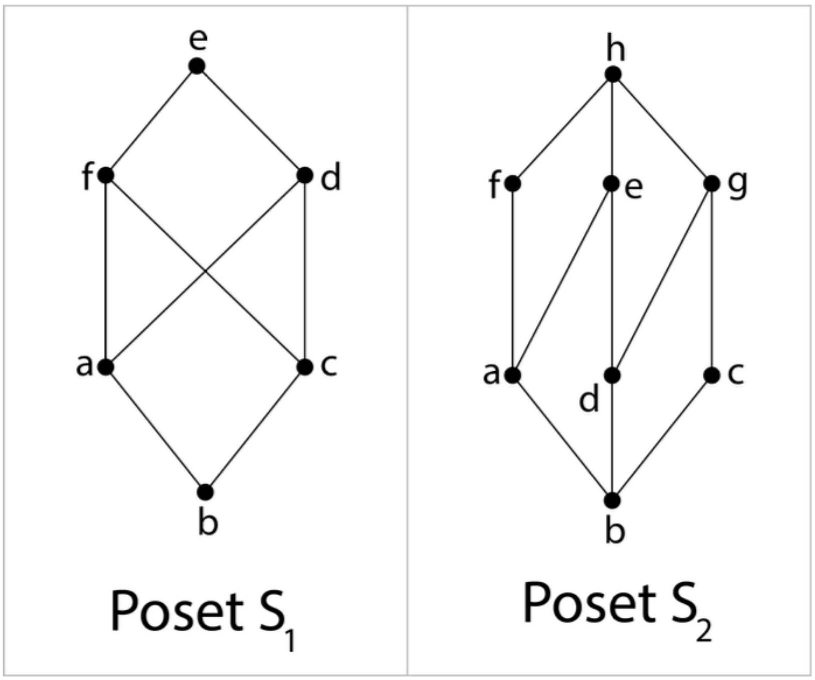
17. Let $H = \langle 24 \rangle$ be a cyclic subgroup of $\mathbb{Z}_{105}$. Explain why H is isomorphic to $\mathbb{Z}_m$ for some integer m, and find the appropriate m that works. (*Hint:* Start by computing $|H|$ using the appropriate result from section 15.1.)

18. Consider posets S_1 and S_2 whose Hasse diagrams are given below.

(a) For each poset, find the greatest and least elements, if they exist.

(b) For each poset, find $a \vee c$ and $a \wedge c$, if they exist.

(c) Is the poset S_1 a lattice? Explain your answer using the definition of lattice.



19. (a) Is the set $H = \langle 3 \rangle \times \langle 5 \rangle$ a cyclic subgroup of $\mathbb{Z}_6 \times \mathbb{Z}_{10}$? If it is, find a generator. If it's not, explain why not.

(b) Same as the previous part, but with $H = \langle 4 \rangle \times \langle 5 \rangle$.

20. Let $f = (1357)(8642)$, $g = (12)(38)(47)(56)$, and $h = (1248)$ be elements of the group S_8.

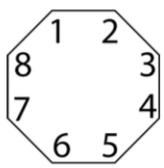
(a) Compute the cyclic subgroup generated by f. Then find the order of f.

(b) Compute f^{55}. (*Hint:* Use the previous part.)

(c) Compute f^{-1}.

(d) Compute $h \circ f \circ h^{-1}$. Leave your answers as a product of disjoint cycles.

(e) D_8 is the set of symmetries of the octagon below. Is f an element of D_8? If so, give a geometric description of f as a rotation or reflection.



(f) Same as the previous part, but with the permutation g.

21. Consider the following linear code:

The generator matrix is $G = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$, and the parity-check matrix is $P = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

For this code, the original messages are bit strings of length 3, and the code can correct up to one error.

(a) Compute the code word corresponding to the message $x = (0, 1, 1)$.

(b) Assume that at most one error occurs when a code word is transmitted and received. Suppose that the message $(1, 0, 1, 0, 0, 0)$ is received. Compute the syndrome, then use it to determine the code word that was sent.

Good luck on the exam, and have a great summer!