Discrete Structures II - Topics for Spring 2026

Notes:

- "Know basic terminology" means that students should have at least a superficial understanding of the given terminology. For example, this might mean that they can identify when a given object satisfies a definition or doesn't (at least for sufficiently simple examples). Also, they should be able to justify their answers in a reasonable way.

- Students should know the standard binary operations for the following groups: $\mathbb{Z}_n$, $\mathbb{U}_n$, $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{R}^*$, and $\mathbb{R}^+$.

  For the basic terminology in the group theory chapters, students should be able to do computations for these groups.

Section 9.1: Graphs - General Introduction

- Know basic terminology: Directed / undirected graphs, vertices, edges, degree in undirected graphs, in-degree / out-degree in a directed graph, paths, circuits, subgraphs, spanning subgraph, complete graph K_n.
- Graph isomorphism: For two isomorphic graphs, be able to give an example of an isomorphism. If two graphs are not isomorphic, explain why an isomorphism can't exist.

Section 9.2: Data Structures for Graphs

- Adjacency matrix: Be able to write down the adjacency matrix for a graph. Given an adjacency matrix, draw the corresponding graph.

Section 9.3: Connectivity

- Know what connected / strongly connected means.
- Identify the connected components in a graph.
- Be able to use the breadth-first search in Algorithm 9.3.8 to find a path between vertices in a graph.

Section 9.4: Traversals: Eulerian and Hamiltonian Graphs

- Use Euler's Theorem (Theorem 9.4.7) to determine if an Eulerian path and/or circuit exists.
- Be able to find Eulerian paths and circuits in a graph when they exist.
- Be able to find a Hamiltonian paths or circuits in small graphs when they exist.

Section 9.6: Planarity and Graph Colorings

- Know basic terminology: planar graph, planar embedding, graph coloring, n-coloring, bipartite graph.
- Given a graph, find the chromatic number of the graph, and be able to explain why the answer is correct.

Section 10.1: What Is a Tree?

- Know basic terminology: cycle, tree.
- Know the relationship between the number of vertices and edges in a tree.

Section 10.2: Spanning Trees

- Know basic terminology: Spanning tree, minimal spanning tree.
- In a weighted graph, compute the weight of a spanning tree.
- Be able to use Prim's algorithm to find a minimal spanning tree.

Section 10.3: Rooted Trees

- Know basic terminology: rooted tree, root, parent, child, ancestors, descendants, subtrees, level of a vertex, depth.

Section 10.4: Binary Trees

- Know basic terminology: Ordered rooted trees, binary trees, left / right subtrees, leaf, internal vertices.
- Be to able to perform preorder, inorder, and postorder traversals of binary trees.
- Write the binary sort tree for a list of numbers. The inorder traversal yields the list of numbers in increasing order.
- Write down the expression tree for an algebraic expression.

Section 11.1: Operations

- Know basic terminology: Unary operation, binary operation.
- Know the properties of operations: Commutative, associative, identity, inverse, idempotent, left / right distributive, distributive, involution, closure.
- Be able to identify and explain if an operation has a certain property or not.
- Understand and be able to construct operation tables.

Section 11.2: Algebraic Systems

- Know basic terminology: groups and basic notation for groups, abelian group.
- Given a set with a binary operation, determine if it's a group or not. If it is, prove it. If not, explain why not.

Section 11.3: Some General Properties of Groups

- Know the basic properties of identity and inverses (Theorems 11.3.1-9). Be able to prove the properties using the definition of group.
- Understand appropriate notation for inverses and exponentiation in additive groups. (See Example 11.3.4(d) and Example 11.3.12(b) in the textbook.)
- Compute powers of elements of groups, and be able to use the basic properties given by Lemma 11.3.13 and Theorem 11.3.14.

Section 11.4: Greatest Common Divisors and the Integers Modulo n

- Do computations using modular arithmetic.
- Understand that $\mathbb{Z}_n$ is a group under addition and $\mathbb{U}_n$ is a group under multiplication. Be able to do computations with these groups, including the group operations, exponentiation, and finding inverses.
- Use the Euclidean algorithm to find the greatest common divisor of two positive integers.

Section 11.5: Subsystems

- Use conditions (Theorem 11.5.3 / 11.5.5) for checking if a subset of a group is a subgroup.
- Given an element of a group, compute the cyclic group generated by that element.
- Determine if a subgroup is a cyclic group.
- Compute the order of an element in a group.

Section 11.6: Direct Products

- Do computations involving direct products of groups, including finding the identity element, inverses, and powers of elements.
- Be able to determine if a subset of a direct product is a subgroup.

Section 11.7: Isomorphisms

- Given two groups that are isomorphic, give an isomorphism and prove that it works using the definition.
- If two groups aren't isomorphic, explain why not.

Section 13.1: Posets Revisited

- Given a poset, be able to draw the Hasse diagram.
- Given a poset, be able to find lower / upper bounds, greatest lower bound, least upper bound, greatest element, and least element. In particular, you should be able to compute these objects for the following examples of posets:
    - A set of integers with divides relation
    - The power set $\mathcal{P}(S)$ with subset relation

Section 13.2: Lattices

- Know basic terminology: Join / meet, lattice, distributive lattice
- Determine join and meet for any two elements of the lattice.
- Determine if a given poset is a lattice.

Section 15.1: Cyclic Groups

- Know basic terminology: Cyclic group definition.
- Be able to determine if a group is cyclic or not.
- Know possible structures for a cyclic group (Theorem 15.1.9), the structure of subgroups (Theorem 15.1.10), and the order of elements in a cyclic group (Theorem 15.1.10).
- Given a cyclic group: Be able to give an isomorphism from $\mathbb{Z}$ or $\mathbb{Z}_n$ to the cyclic group.
- Compute the order of elements in a cyclic group.
- State the types of subgroups that could occur in terms of $\mathbb{Z}$ or $\mathbb{Z}_n$.

Section 15.3: Permutation Groups

- Use notation for permutations: Matrix form and cycle notation.
- Express permutations as products of disjoint cycles.
- Find inverses and compositions of permutations.
- Know basic terminology: Symmetric group S_n, dihedral group D_n.
- Know how elements of D_n represent symmetries of regular n-gons. Be able to find inverses and compositions of elements in D_n, and describe the results geometrically.

Section 15.5: Coding Theory, Linear Codes

- For error correcting codes:
  - Know basic terminology: Code word, generator matrix, parity-check matrix, syndrome.
  - Know how to encode / decode messages.