

Math 475, Problem Set #10  
(due 4/13/06)

- A. Consider the sequence 1, 2, 8, 40, 224, 1344, 8448, 54912, ... defined by the initial condition  $a_1 = 1$  and the recurrence relation  $a_n = 2(a_1a_{n-1} + a_2a_{n-2} + \dots + a_{n-1}a_1)$  (valid for all  $n \geq 2$ ). Find (and prove) a general formula for  $a_n$ .
- B. Chapter 7, problem 22. (Hint: Label the points 1 through  $2n$ . Let  $h_{n,k}$  be the number of ways to join the points in pairs so that the resulting line segments do not intersect, where point 1 is joined to point  $k$ . Show that  $h_{n,k} = 0$  when  $k$  is odd, and find a formula for  $h_{n,k}$  in terms of  $h_1, h_2, \dots, h_{n-1}$  when  $k$  is even. Use this to write  $h_n$  as a sum of products of earlier terms of the sequence.) You may find it convenient to define  $h_0 = 1$ .
- C. Chapter 7, problem 41.
- D. Chapter 7, problem 42(c).
- E. Chapter 7, problem 44.
- F. Chapter 7, problem 46.