

Math 475, Problem Set #11  
(due 4/20/06)

- A. Chapter 8, problem 2.
- B. Find (and prove) a formula for the number of integer sequences  $a_1, a_2, \dots, a_n$  with  $1 \leq a_1 \leq a_2 \leq \dots \leq a_n \leq n$  and  $a_k \geq k$  for all  $k$ .
- C. Repeat the Example from the middle of page 270, but this time assume that the cash register starts with a single 50 cent piece (rather than starting empty). We still assume that there are  $2n$  people in line to get into the theatre, that admission costs 50 cents, that  $n$  of the people in line have a 50 cent piece and  $n$  of them have a 1 dollar bill. In how many ways can the people line up so that whenever a person with a 1 dollar bill buys a ticket, the box office has a 50 cent piece in order to make change?
- D. Chapter 8, problem 7. Express both  $h_n$  and  $\sum_{k=0}^n h_k$  as polynomials in  $n$  in the ordinary way.
- E. Chapter 8, problem 8. Express  $\sum_{k=1}^n k^5$  as a polynomial in  $n$  in the ordinary way.

By the way (maybe I shouldn't have to say this, but it can't hurt): For problems D and E, a good way to make sure that you haven't made some arithmetical error en route to your final answer is to check it by plugging in suitable values of  $n$ . I am often surprised by how few students take the time to do this, even though it's a quick way to catch mistakes and thereby boost your homework score.