Math 475, Problem Set #5
(due 2/23/06)

A. Chapter 3, problem 28. Do part (a) in two different ways: once by brute force (i.e., dynamic programming), and once by interpreting the counting of routes in terms of multiset permutations. Likewise, do part (b) in two different ways: once by dynamic programming, and once by multiset permutations (making use of Brualdi's hint as well). You may use a calculator or computer to facilitate the dynamic programming computation.

B. Chapter 3, problem 40.

C. (a) Chapter 3, problem 48. Do this problem directly in terms of multiset permutations. (Hint: Look at the special case $m = n = 2$. What reversible operation might you perform on a string of 3 A's and 2 B's that would turn it into a string of 2 A's and at most 2 B's?)

(b) Use the addition principle (just once) to show that

$$p(m,m)+p(m+1,m)+p(m+2,m)+\ldots+p(m+n,m) = p(m+n+1,m+1),$$

where $p(\cdot,\cdot)$ is as section 5.1.

(c) Explain the relationship between parts (a) and (b) of this problem.

D. Chapter 3, problem 49. Find and fix Brualdi's mistake. (Hint: Look at the special case $m = n = 1$. What reversible operation might you perform on a string of 2 A's and 2 B's that would turn it into a string of at most 1 A and at most 1 B? If you're stuck for ideas, take another look at part (a) of the preceding problem!)

E. Let $f(n)$ be the nth Fibonacci number, so that $f(1) = 1$, $f(2) = 2$, and $f(n) = f(n-1) + f(n-2)$ for all $n \geq 3$. Prove by induction that the sum $f(1) + f(2) + \ldots + f(n)$ is equal to $f(n+2) - 2$, for all $n \geq 1$.