Math 475, Problem Set #6
(due 3/2/06)

A. (a) For each point (a, b) with a, b non-negative integers satisfying $a+b \leq 8$, count the paths from (0,0) to (a, b) where the legal steps from (i, j) are to $(i+2, j)$, $(i, j+2)$, and $(i+1, j+1)$.

(b) Compute the coefficients of $(x^2 + x + 1)^n$ for $n = 0, 1, 2, 3, 4$.

(c) Based on parts (a) and (b), formulate a precise conjecture of the form "for all non-negative integers a and b, the number of paths from $(0, 0)$ to (a, b) is equal to the coefficient of ... in the polynomial ...".

B. Chapter 5, problem 12.

C. Solve Brualdi, Chapter 5, problem 18 in two different ways: once using problem 16 as a model, and once using problem 17 as a model.

D. What is the coefficient of $x_1^3 x_2^3 x_3 x_4^2$ in the expansion of $(x_1 - x_2 + 2x_3 - 2x_4)^9$?

E. Brualdi, Chapter 5, problem 46. Retain all terms that are greater than 10^{-3}; discard the rest.

F. Fix positive integers $n, k \geq 3$. Consider a convex n-gon with vertices labelled 1 through n. Call a convex k-gon, whose vertices are a subset of the vertices of the n-gon, an *internal k-gon* if all of its sides are diagonals of the n-gon.

(a) How many internal k-gons are there containing the vertex labelled 1?

(b) How many internal k-gons are there all together? (Hint: What do you know ahead of time about the ratio between the answer to (a) and the answer to (b)?)