Math 491, Problem Set #1
(due 9/11/03)

1. (a) Write and run a program to compute $f(n) = \sum_{k=0}^{n}(-1)^k\binom{n}{k}^2$. (Submit this part by email to `propp@math.harvard.ee-dee-you`.)

   (b) Devise a conjecture about the value of $f(n)$.

   (c) Prove your conjecture using the algebraic interpretation of $\binom{n}{k}$ as the coefficient of x^k in $(1+x)^n$.

   (d) Prove your conjecture using the interpretation of $\binom{n}{k}$ as the number of combinations of n things taken k at a time.

2. Define a sequence of functions $f_0(x), f_1(x), f_2(x), ...$ where $f_0(x) = x$, $f_1(x) = x$, and for all $n > 1$, $f_n(x) = ([f_{n-1}(x)]^2 + 1)/f_{n-2}(x)$. Thus, $f_2(x) = x + x^{-1}$, $f_3(x) = x + 3x^{-1} + x^{-3}$, etc.

   (a) Formulate a conjecture about the values of $f_n(1)$.

   (b) Formulate a conjecture about the values of $f_n(-1)$.

   (c) Formulate a conjecture about the values of $f_n(i)$, where $i = \sqrt{-1}$.

In each case, if you can't get all the way through, explain how far you got and what the obstacles were.

For each problem on this assignment (and in every future assignment!), please be sure to write down how many hours you spent working on the problem, and whom you worked on it with. Also keep in mind that having the correct answer or main idea is not enough; it must be expressed clearly.