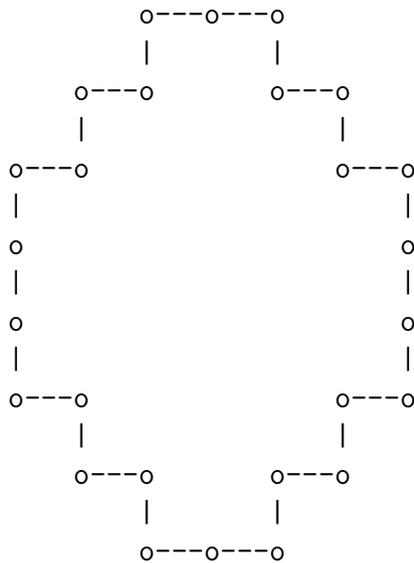


Math 491, Problem Set #13  
(due 11/11/01)

1. An “augmented Aztec diamond of order  $n$ ” is a subset of the square grid formed by symmetrically stacking rectangles of height 1 and respective widths  $2, 4, 6, \dots, 2n - 2, 2n, 2n, 2n, 2n - 2, \dots, 6, 4, 2$  (note that there are 3 rectangles of width  $2n$  being stacked). Here, for instance, is an augmented Aztec diamond of order 3:



Express the generating function  $\sum_{n \geq 0} a_n t^n$  as an algebraic function of  $t$ , where  $a_n$  denotes the number of domino-tilings of the augmented Aztec diamond of order  $n$ , and we put  $a_0 = 1$ . (Note that the augmented Aztec diamond of order 1 is just a rectangle of height 3 and width 2.)

2. Use the exchange principle for 2-routings to find a compact formula for the number of lozenge-tilings of the semiregular hexagon with side-lengths  $a, b, 2, a, b, 2$ .