Math 491, Problem Set #13
(due 11/11/01)

1. An "augmented Aztec diamond of order n" is a subset of the square grid formed by symmetrically stacking rectangles of height 1 and respective widths $2, 4, 6, \ldots, 2n-2, 2n, 2n, 2n, 2n-2, \ldots, 6, 4, 2$ (note that there are 3 rectangles of width $2n$ being stacked). Here, for instance, is an augmented Aztec diamond of order 3:

```
        o---o---o
        |       |
    o---o       o---o
    |               |
o---o               o---o
|                       |
o                       o
|                       |
o                       o
|                       |
o---o               o---o
    |               |
    o---o       o---o
        |       |
        o---o---o
```

   Express the generating function $\sum_{n \geq 0} a_n t^n$ as an algebraic function of t, where a_n denotes the number of domino-tilings of the augmented Aztec diamond of order n, and we put $a_0 = 1$. (Note that the augmented Aztec diamond of order 1 is just a rectangle of height 3 and width 2.)

2. Use the exchange principle for 2-routings to find a compact formula for the number of lozenge-tilings of the semiregular hexagon with side-lengths $a, b, 2, a, b, 2$.