Math 491, Problem Set #15
(due 11/18/03)

(a) Let A_n be the average number of times that a $2n$-step Dyck path returns to the origin (counting $(2n, 0)$ as a return but not $(0, 0)$), so that $A_0 = 0$, $A_1 = 1$, $A_2 = 3/2$, and $A_3 = 9/5$. Use Maple to compute A_n for various small values of n (1 through 6, at least), and conjecture a general formula.

(b) Give an algebraic proof of your conjecture using generating functions.

(c) Give a bijective proof of your conjecture, using the relationship between Dyck paths and triangulations of polygons.