Math 491, Problem Set #5
(due 9/25/03)

Let a_n be the number of domino tilings of a 3-by-$2n$ rectangle, and let b_n be the number of domino tilings of a 3-by-$(2n+1)$ rectangle from which a corner square has been removed. We showed in class that $a_n = a_{n-1} + 2b_{n-1}$ and $b_n = a_{n-1} + 3b_{n-1}$ for all $n \geq 1$.

1. Introduce
$$A(x) = a_0 + a_1 x + a_2 x^2 + \ldots$$
and
$$B(x) = b_0 + b_1 x + b_x x^2 + \ldots .$$
Write down two algebraic relations between $A(x)$ and $B(x)$ that represent the two recurrence relations (taking care to incorporate the boundary conditions correctly), and solve for $A(x)$ and $B(x)$.

2. We also saw in class that
$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}^n \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$
Use linear algebra to derive a formula for a_n.

Please be sure to write down how many hours you spent working on the assignment, and whom you worked with.