Math 431, Assignment #5

(due 3/22/01)

This assignment covers sections 4.2, 4.6, and 4.7. Most of the problems come from pages 173-188 of Ross.

Remember, as on the previous assignments, to indicate how many hours you spent on the assignment, and whom you worked with. Also remember to show your work if you want to receive credit.

1. As in the last problem of the preceding problem set, suppose I roll three six-sided dice, where the first die has two faces marked "1" and four faces marked "4", the second die is marked like the first one, and the third die has four faces marked "2" and two faces marked "5". Assume the rolls of the three dice are independent, and let X, Y, and Z denote the numbers shown by the respective dice. Compute the variance and standard deviation of each of the following random variables: X, Y, Z, $X+Y$, $X+Z$. (For the last two, do not make use of the theorem about the variance of a sum of two independent random variables; do the calculation "honestly".)

2. I have two spinners to choose from. Spinner A yields the value 1 half of the time and the value 4 half of the time; spinner B yields the value 2 half of the time and the value 3 half of the time. I play a game in which I pick one of the spinners, spin it five times, and take the sum of the numbers that I spin. Call this sum X.

   (a) Compute $E(X)$ and $\mathrm{Var}(X)$ if I choose spinner A. Do the same for spinner B.

   (b) Suppose I'll win a dollar if X is 14 or larger. For each of the two spinners, calculate the probability that I'll win. Which spinner should I pick if I want to make my probability of winning as large as possible?

   (c) Now suppose I'll win a dollar if X is 11 or larger. For each of the two spinners, calculate the probability that I'll win. Which spinner should I pick if I want to make my probability of winning as large as possible?

3. Twelve passengers enter an elevator at the basement and independently choose to exit randomly at one of the ten above-ground floors. What is the expected number of stops that the elevator will have to make? (Hint: Write the random variable as a sum of ten indicator random variables.)

4. Let α be some positive real number. Suppose X is a random variable such that for all $n \geq 1$, $P(X = \alpha^n) = (\frac{1}{2})^n$ (this is a bona fide probability mass function since $\sum_{n=1}^{\infty} (\frac{1}{2})^n = 1$).

   (a) For what values of α is $E(X)$ finite?

   (b) For what values of α is $E(X^2)$ finite?

   (c) Use parts (a) and (b) to give an example of a random variable whose mean is finite but whose variance is infinite.

5. Problem 4.33.

6. Problem 4.43.

7. Problems 4.67 and 4.68.

8. Problem 4.69.

9. Theoretical exercise 4.9 (page 185).

10. Theoretical exercise 4.13 (page 186).

Each problem is worth 10 points. Additionally, you can get up to 5 bonus points for making a good estimate of your raw score (which will lie between 0 and 100).