Math 567, Assignment #11

(due Thursday, May 4)

This assignment covers material discussed in class on April 27. If you missed class, contact me and I'll catch you up. Some of this material also appears in the chapters of Burton concerning Fibonacci numbers and continued fractions (chapters 13 and 14).

Note that for both problems, you are supposed to use the techniques discussed in class; brute-force solutions will not be awarded points.

1. A (positive) square cannot be twice a square, since the square root of 2 is irrational. However, a (positive) triangle number can be twice a triangle number (witness the triangle numbers 6 and 3). Give two other pairs of (positive) triangle numbers, with the larger equal to twice the smaller, and show that infinitely many such pairs of triangle numbers exist.

2. Find three pairs of positive integers (a, b) such that $a^2 - 3ab - b^2 = 1$, and show that there are infinitely many more. (Hint: take the convergents arising from the continued fraction expansion of the positive root of $x^2 - 3x - 1 = 0$.)