Math 567, Assignment #2

(due February 10)

1. For what moduli n is it true that $a^2 \equiv b^2 \pmod{n}$ implies $a \equiv b \pmod{n}$?

2. Find the remainders when 2^{50} and 41^{65} are divided by 7.

3. Use congruence calculations to show that $2^{37} - 1$ is divisible by 223. All calculations should be done entirely by hand, and should not involve any numbers greater than one million.

4. What is the remainder when the following sum is divided by 4?

$$1^5 + 2^5 + 3^5 + \ldots + 100^5$$

5. Prove that if $a_1, a_2, ..., a_n$ is a complete set of residues modulo n then $c + a_1, c + a_2, ..., c + a_n$ is also a complete set of residues modulo n.

6. (a) Prove that if $a_1, a_2, ..., a_n$ is a complete set of residues modulo n and $\gcd(c, n) = 1$, then $ca_1, ca_2, ..., ca_n$ is also a complete set of residues modulo n. (b) Show that the hypothesis on the gcd is mandatory by showing that if $\gcd(c, n) \neq 1$, then the claim is false for *every* complete set of residues $a_1, a_2, ..., a_n$.

7. Prove or disprove: If $a_1, a_2, ..., a_5$ is a complete set of residues modulo 5, then so is $a_1^3, a_2^3, ..., a_5^3$.

8. Find the last two digits of the number 9^{9^9} (which equals $9^{(9^9)}$ by convention).