Math 567, Assignment #3

(due February 17)

1. Solve each of the following sets of simultaneous congruences:
(a) $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$.
(b) $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$.
(c) $x \equiv 5 \pmod 6$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$.
(d) $2x \equiv 1 \pmod 5$, $3x \equiv 9 \pmod 6$, $4x \equiv 1 \pmod 7$, $5x \equiv 9 \pmod{11}$.

2. Find three consecutive integers, the first of which is divisible by a square, the second by a cube, and the third by a fourth power.

3. When eggs in a basket are removed 2 at a time, 1 remains. When eggs are removed 3 at a time, 2 remain. When eggs are removed 4 at a time, 3 remain. When eggs are removed 5 at a time, 4 remain. When eggs are removed 6 at a time, 5 remain. But: when eggs are removed 7 at a time, none remain. Find the smallest number of eggs that could have been in the basket.

4. If $x \equiv a \pmod n$, prove that either $x \equiv a \pmod{2n}$ or $x \equiv a + n \pmod{2n}$.

5. Obtain all the incongruent solutions modulo 210 of the system

$$2x \equiv 3 \pmod 5$$

$$4x \equiv 2 \pmod 6$$

$$3x \equiv 2 \pmod 7$$

6. Solve the following system of congruences:

$$x + 2y \equiv 3 \pmod 7$$

$$4x + 5y \equiv 6 \pmod 7$$

(Hint: Try to mimic the steps you'd use in solving this equation in the ordinary way.)

7. Show that there must exist infinitely many primes that end in 3 or 7.