Math 567, Assignment #5

(due March 2)

This assignment covers material up through section 6.1.

1. Use Fermat's theorem to show that every prime divisor of $2^{32}+1$ must be congruent to 1 (mod 64), and then find a prime of this form that divides $2^{32}+1$, thereby proving that this number is composite.

2. Use Fermat's theorem to find a prime factor of $6^{16}+1$.

3. State and prove formulas that express:
(a) the integer $(2p)!/p^2$ modulo p;
(b) the integer $(3p)!/p^3$ modulo p;
(c) the integer $(p^2)!/p^{p+1}$ modulo p; and
(d) the integer $(1 \cdot 3 \cdot 5 \cdot 7 \cdot ... \cdot (p-2))^2$ modulo p (with p odd).

4. Which of the following functions of n are multiplicative?
(a) $f(n)$ = the sum of the prime divisors of n.
(b) $g(n)$ = the sum of the square divisors of n.
(c) $h(n)$ = the number of solutions modulo n to the congruence $x^2 \equiv -1 \pmod{n}$.