

In[16] := **byfirsts**[2]

Out[16] = {{0, 0}, {0, 1}}

In[17] := **byfirsts**[3]

Out[17] = {{{-1, 2 +  $\frac{1}{Y}$ }, {-1, 2}, {-1, 2 +  $\frac{1}{Y}$  + Y}, {-1, 2 + Y}}, {{0, 0}, {0, 1}, {0, 2}}}

In[18] := **byfirsts**[4]

Out[18] = {{{-2, 3 +  $\frac{2}{Y}$ }, {-2, 3 +  $\frac{1}{Y}$ }, {-2, 3}, {-2, 3 +  $\frac{2}{Y}$  + Y},  
 {-2, 3 +  $\frac{1}{Y}$  + Y}, {-2, 3 + Y}, {-2, 3 +  $\frac{2}{Y}$  + 2Y}, {-2, 3 +  $\frac{1}{Y}$  + 2Y}, {-2, 3 + 2Y}},  
 {{{-1, 2 +  $\frac{1}{Y}$ }, {-1, 2}, {-1, 3 +  $\frac{1}{Y}$ }, {-1, 3}, {-1, 2 +  $\frac{1}{Y}$  + Y}, {-1, 2 + Y},  
 {-1, 3 +  $\frac{1}{Y}$  + Y}, {-1, 3 + Y}}, {{0, 0}, {0, 1}, {0, 2}, {0, 3}},  
 {{{-2 -  $\frac{1}{X}$ , 3 +  $\frac{1}{Y^2}$  +  $\frac{2}{Y}$ }, {-2 -  $\frac{1}{X}$ , 3 +  $\frac{2}{Y}$ }, {-2 -  $\frac{1}{X}$ , 3 +  $\frac{1}{Y^2}$  +  $\frac{2}{Y}$  + Y},  
 {-2 -  $\frac{1}{X}$ , 3 +  $\frac{2}{Y}$  + Y}, {-2 -  $\frac{1}{X}$ , 3 +  $\frac{1}{Y^2}$  +  $\frac{2}{Y}$  + 2Y}, {-2 -  $\frac{1}{X}$ , 3 +  $\frac{2}{Y}$  + 2Y}},  
 {{{-2 - X, 3 +  $\frac{2}{Y}$  + 2Y}, {-2 - X, 3 +  $\frac{1}{Y}$  + 2Y}, {-2 - X, 3 + 2Y},  
 {-2 - X, 3 +  $\frac{2}{Y}$  + 2Y + Y^2}, {-2 - X, 3 +  $\frac{1}{Y}$  + 2Y + Y^2}, {-2 - X, 3 + 2Y + Y^2}},  
 {{{-2 -  $\frac{1}{X}$  - X, 4 +  $\frac{2}{Y}$  + 2Y}, {-2 -  $\frac{1}{X}$  - X, 4 +  $\frac{1}{Y^2}$  +  $\frac{2}{Y}$  + 2Y}, {-2 -  $\frac{1}{X}$  - X, 3 +  $\frac{1}{Y^2}$  +  $\frac{2}{Y}$  + 2Y},  
 {-2 -  $\frac{1}{X}$  - X, 3 +  $\frac{2}{Y}$  + 2Y}, {-2 -  $\frac{1}{X}$  - X, 3 +  $\frac{1}{Y^2}$  +  $\frac{2}{Y}$  + 2Y + Y^2}, {-2 -  $\frac{1}{X}$  - X, 3 +  $\frac{2}{Y}$  + 2Y + Y^2},  
 {-2 -  $\frac{1}{X}$  - X, 4 +  $\frac{2}{Y}$  + 2Y + Y^2}, {-2 -  $\frac{1}{X}$  - X, 4 +  $\frac{1}{Y^2}$  +  $\frac{2}{Y}$  + 2Y + Y^2}}}

In[19] := **byfirsts**[5]

Out[19] = {{{-3, 4 +  $\frac{2}{Y}$ }, {-3, 4 +  $\frac{3}{Y}$ }, {-3, 4 +  $\frac{1}{Y}$ }, {-3, 4}, {-3, 4 +  $\frac{2}{Y}$  + Y}, {-3, 4 +  $\frac{3}{Y}$  + Y},  
 {-3, 4 +  $\frac{1}{Y}$  + Y}, {-3, 4 + Y}, {-3, 4 +  $\frac{2}{Y}$  + 2Y}, {-3, 4 +  $\frac{3}{Y}$  + 2Y}, {-3, 4 +  $\frac{1}{Y}$  + 2Y},  
 {-3, 4 + 2Y}, {-3, 4 +  $\frac{2}{Y}$  + 3Y}, {-3, 4 +  $\frac{3}{Y}$  + 3Y}, {-3, 4 +  $\frac{1}{Y}$  + 3Y}, {-3, 4 + 3Y}},  
 {{{-2, 3 +  $\frac{2}{Y}$ }, {-2, 3 +  $\frac{1}{Y}$ }, {-2, 3}, {-2, 4 +  $\frac{2}{Y}$ }, {-2, 4 +  $\frac{1}{Y}$ }, {-2, 4}, {-2, 3 +  $\frac{2}{Y}$  + Y},  
 {-2, 3 +  $\frac{1}{Y}$  + Y}, {-2, 3 + Y}, {-2, 4 +  $\frac{2}{Y}$  + Y}, {-2, 4 +  $\frac{1}{Y}$  + Y}, {-2, 4 + Y}, {-2, 4 +  $\frac{2}{Y}$  + 2Y},  
 {-2, 4 +  $\frac{1}{Y}$  + 2Y}, {-2, 4 + 2Y}, {-2, 3 +  $\frac{2}{Y}$  + 2Y}, {-2, 3 +  $\frac{1}{Y}$  + 2Y}, {-2, 3 + 2Y}},  
 {{{-1, 2 +  $\frac{1}{Y}$ }, {-1, 2}, {-1, 3 +  $\frac{1}{Y}$ }, {-1, 3}, {-1, 4 +  $\frac{1}{Y}$ }, {-1, 4}, {-1, 2 +  $\frac{1}{Y}$  + Y},  
 {-1, 2 + Y}, {-1, 3 +  $\frac{1}{Y}$  + Y}, {-1, 3 + Y}, {-1, 4 +  $\frac{1}{Y}$  + Y}, {-1, 4 + Y}},  
 {{0, 0}, {0, 1}, {0, 2}, {0, 3}, {0, 4}},  
 {{{-3 -  $\frac{2}{X}$ , 4 +  $\frac{2}{Y^2}$  +  $\frac{3}{Y}$ }, {-3 -  $\frac{2}{X}$ , 4 +  $\frac{3}{Y}$ }, {-3 -  $\frac{2}{X}$ , 4 +  $\frac{1}{Y^2}$  +  $\frac{3}{Y}$ },  
 {-3 -  $\frac{2}{X}$ , 4 +  $\frac{2}{Y^2}$  +  $\frac{3}{Y}$  + Y}, {-3 -  $\frac{2}{X}$ , 4 +  $\frac{3}{Y}$  + Y}, {-3 -  $\frac{2}{X}$ , 4 +  $\frac{1}{Y^2}$  +  $\frac{3}{Y}$  + Y}}











$$\begin{aligned}
& \left\{ -3 - \frac{1}{x^2} - \frac{2}{x} - 2x - x^2, 4 + \frac{1}{y^3} + \frac{2}{y^2} + \frac{3}{y} + 3y + 2y^2 + y^3 \right\}, \\
& \left\{ -3 - \frac{1}{x^2} - \frac{2}{x} - 2x - x^2, 4 + \frac{2}{y^2} + \frac{3}{y} + 3y + 2y^2 + y^3 \right\}, \\
& \left\{ -3 - \frac{1}{x^2} - \frac{2}{x} - 2x - x^2, 5 + \frac{1}{y^3} + \frac{2}{y^2} + \frac{3}{y} + 3y + 2y^2 + y^3 \right\}, \\
& \left\{ -3 - \frac{1}{x^2} - \frac{2}{x} - 2x - x^2, 5 + \frac{2}{y^2} + \frac{3}{y} + 3y + 2y^2 + y^3 \right\}, \\
& \left\{ -3 - \frac{1}{x^2} - \frac{2}{x} - 2x - x^2, 6 + \frac{1}{y^3} + \frac{2}{y^2} + \frac{3}{y} + 3y + 2y^2 + y^3 \right\}, \\
& \left\{ -3 - \frac{1}{x^2} - \frac{2}{x} - 2x - x^2, 6 + \frac{2}{y^2} + \frac{3}{y} + 3y + 2y^2 + y^3 \right\} \}, \\
& \left\{ \left\{ -3 - \frac{1}{x} - 2x - x^2, 4 + \frac{2}{y} + 3y + 2y^2 \right\}, \left\{ -3 - \frac{1}{x} - 2x - x^2, 4 + \frac{3}{y} + 3y + 2y^2 \right\}, \right. \\
& \left. \left\{ -3 - \frac{1}{x} - 2x - x^2, 4 + \frac{1}{y^2} + \frac{3}{y} + 3y + 2y^2 \right\}, \left\{ -3 - \frac{1}{x} - 2x - x^2, 4 + \frac{1}{y^2} + \frac{2}{y} + 3y + 2y^2 \right\}, \right. \\
& \left. \left\{ -3 - \frac{1}{x} - 2x - x^2, 5 + \frac{1}{y^2} + \frac{3}{y} + 3y + 2y^2 \right\}, \left\{ -3 - \frac{1}{x} - 2x - x^2, 5 + \frac{3}{y} + 3y + 2y^2 \right\}, \right. \\
& \left. \left\{ -3 - \frac{1}{x} - 2x - x^2, 5 + \frac{2}{y} + 3y + 2y^2 \right\}, \left\{ -3 - \frac{1}{x} - 2x - x^2, 5 + \frac{1}{y^2} + \frac{2}{y} + 3y + 2y^2 \right\}, \right. \\
& \left. \left\{ -3 - \frac{1}{x} - 2x - x^2, 4 + \frac{2}{y} + 3y + 2y^2 + y^3 \right\}, \left\{ -3 - \frac{1}{x} - 2x - x^2, 4 + \frac{3}{y} + 3y + 2y^2 + y^3 \right\}, \right. \\
& \left. \left\{ -3 - \frac{1}{x} - 2x - x^2, 4 + \frac{1}{y^2} + \frac{3}{y} + 3y + 2y^2 + y^3 \right\}, \left\{ -3 - \frac{1}{x} - 2x - x^2, 4 + \frac{1}{y^2} + \frac{2}{y} + 3y + 2y^2 + y^3 \right\}, \right. \\
& \left. \left\{ -3 - \frac{1}{x} - 2x - x^2, 5 + \frac{1}{y^2} + \frac{3}{y} + 3y + 2y^2 + y^3 \right\}, \left\{ -3 - \frac{1}{x} - 2x - x^2, 5 + \frac{3}{y} + 3y + 2y^2 + y^3 \right\}, \right. \\
& \left. \left\{ -3 - \frac{1}{x} - 2x - x^2, 5 + \frac{2}{y} + 3y + 2y^2 + y^3 \right\}, \left\{ -3 - \frac{1}{x} - 2x - x^2, 5 + \frac{1}{y^2} + \frac{2}{y} + 3y + 2y^2 + y^3 \right\} \right\}
\end{aligned}$$

In[20] := **firsts**[5]

$$\begin{aligned}
\text{Out}[20] = & \left\{ -3, -2, -1, 0, -3 - \frac{2}{x}, -3 - \frac{1}{x^2} - \frac{2}{x}, -3 - \frac{1}{x}, -2 - \frac{1}{x}, -3 - 2x, -4 - \frac{2}{x} - 2x, \right. \\
& -3 - \frac{2}{x} - 2x, -4 - \frac{1}{x^2} - \frac{2}{x} - 2x, -3 - \frac{1}{x^2} - \frac{2}{x} - 2x, -3 - \frac{1}{x} - 2x, -3 - x, -2 - x, \\
& -3 - \frac{2}{x} - x, -3 - \frac{1}{x^2} - \frac{2}{x} - x, -3 - \frac{1}{x} - x, -2 - \frac{1}{x} - x, -3 - 2x - x^2, -4 - \frac{2}{x} - 2x - x^2, \\
& \left. -3 - \frac{2}{x} - 2x - x^2, -4 - \frac{1}{x^2} - \frac{2}{x} - 2x - x^2, -3 - \frac{1}{x^2} - \frac{2}{x} - 2x - x^2, -3 - \frac{1}{x} - 2x - x^2 \right\}
\end{aligned}$$