

WHAT CAN NUMBERS BE?: THE FURTHER,
STRANGER ADVENTURES OF PLUS AND TIMES
(book in progress by Jim Propp)

I'm working on a book for the mathematically-interested public, currently called *What Can Numbers Be?: The Further, Stranger Adventures of Plus and Times*. If you think this sounds cool and want to help me make the book better, send me email at jamespropp at gmail dot com.

What's the book about?

It's a travelogue of different number systems (as well as systems involving things we don't call numbers but arguably could): natural numbers, rational numbers, integers, real numbers, complex numbers, polynomials, power series, quaternions, vectors, matrices, modular arithmetic, ideals, p -adic numbers, and finite fields. You could say it's a book about fun things to add and multiply. (If you're thinking "Shouldn't there be chapters about ordinals, cardinals, nonstandard real numbers, and surreal numbers?", you're right! They fit in with the concept and I was originally going to include them, but I've decided to put them in a sequel called *What Can Infinity Be?*.)

Who is the book for?

I'm aiming my book at the kind of adult who felt comfortable with high school mathematics, doesn't know calculus and may be a bit rusty with algebra, but still loves to play with ideas, and who wonders, "What have mathematicians been up to during the past few centuries?"

How will you write it?

I've written about these topics already on my Mathematical Enchantments blog, so I have more than enough material for a book. Most of my blog essays were too long; I always aim for 3000–5000 words and I usually overshoot that target. A lot of what I have to do now is prune. My plan is to take the best of what I've written so far, thread it together using some unifying themes, include more pictures, and make the writing more punchy. I'll try to remove technicalities without oversimplifying, distill mathematical ideas to their essences, and make the prose more readable, using footnotes/endnotes/webnotes to provide more information to those readers who want more details than my typical reader will care for.

What kind of feedback are you looking for?

If you're in my target audience, I want to know: What got you excited? What passages left you lost, or made you feel bored? Which digressions side-tracked your train of attention? Where did you stop reading, and why? (If I could strap readers into fancy equipment that would track eye movements and heart rate and measure galvanic skin response, I would!) The fiction writer Elmore Leonard says that the key to powerful writing is, Don't write the parts that the reader skips. But how will I know what you skipped unless you tell me?

(Here's another way to put it: If you were in a classroom, and I was the teacher, and I was presenting a chapter on the board, at which points would you furtively look around the room to see if other students looked as confused as you felt, or if you felt brave raise your hand and say "Excuse me, but could you explain that last bit to me in more detail?")

If you're a mathematician or historian or philosopher, I want you to keep me honest when my glibness leads to distortion.

If you're anybody, I want you to help me improve my prose. For instance, in my essay on vectors, where I discussed Newton and force, I wrote "In what direction is the force pushing?" and then, in the very next paragraph, I wrote about two people pulling a canal boat. The transition from pushing to pulling could be handled more gracefully. I left this blemish in the blog version of the essay in order to have a ready example of a small way in which my writing could be improved. Hopefully you'll find others that I'm unaware of!

Everyone who helps me will be acknowledged in the published book.

What kind of feedback have you found helpful in the past?

Here are some examples of comments that I've received from pre-readers of my blog essays. You'll notice that they don't mince words, and that's fine with me. If you share my goals for the project, I don't mind feedback that shows me ways in which I've fallen short of my/our goal!

I appreciate that you're trying to find a new way to explain this, but I think a more standard, less creative approach would be better for most readers.

This essay doesn't really seem to have a conclusion or punchline. It sort of ends; I have a hard time saying in one or two sentences what the main message is.

Is that a word everyone knows? I had to google it.

Give an example?

I found this confusing. Had to stop and reread twice.

I got lost (briefly) reading this sentence.

Why undercut your primary message that math is exciting and leads to lots of unexpected, intriguing places using a reinforcement of a negative trope that math just isn't for most folks?

Implicit in the rest of this sentence is the fact that squaring the circle had not yet been proved impossible, but the reader has to figure out that that's what you're saying.

I think your last sentence undersells how freaky and interesting this is.

Why the "housewife" label? Others in this text (male and female) are identified as "amateur mathematicians."

Well, my eyes glazed over, even after several attempts.

I think you're doing the same kind of oversensationalizing that you accuse Singh of doing.

Taken as a whole the article is a little overlong and repetitive. It's not that any one paragraph is irrelevant in isolation; more that this reader starts to feel hit over the head with points that have been adequately made elsewhere. A more ruthless selection of anecdotes might improve the impact.

Do you really want to put politics into an otherwise nonpolitical essay?

Not everyone knows about casting out 9's.

I think these speculations completely disrupt the flow of your essay, and if you just delete the section I quoted, it'll do nothing but improve the essay.

What's your next step in writing the book?

Now that I've written blog-material related to the subjects of nearly all the chapters, I plan to write actual chapter-drafts. Here's a list of the chapters, along with the Mathematical Enchantments ("ME") essays I'll pull material from for the respective chapters (links to all these essay can be found at the Mathematical Enchantments homepage).

1. The Magic Stairway (natural numbers): see ME 11 (Fermat's Last Theorem: the curious incident of the boasting Frenchman), 34 (Who Knows Two?), 82 (The Clatter of the Primes), 91 (The Infinite Stairway), and 92 (Beneath (and Beyond))
2. Dividing When You Can't (rational numbers): see ME 3 (The one about .999...), 22 (More about .999...), and 93 (Denominators and Doppelgänger)

3. The Opposite of a Cow (integers): see ME 15 (Going Negative, part 1), 16 (Going Negative, part 2), 71 (Dividing by Zero), 72 (Who Needs Zero?), 73 (Going Negative, part 3), and 74 (Going Negative, part 4)
4. Holes Too Small to See (real numbers): see ME 51 (My Favorite Theorem), 52 (Calculus is Deeply Irrational), and 94 (Things, Names, and Numbers)
5. Imaginary Friends (complex numbers): see ME 87 (Twisty Numbers for a Screwy Universe)
6. Powers United (polynomials): see ME 88 (Let x Equal x) and 89 (What Lovelace Did: From Bombelli to Bernoulli to Babbage)
7. Unlimited Powers (power series): see ME 95 (Unlimited Powers)
8. The Trouble with Triples (quaternions): see ME 97 (Hamilton's Quaternions, or, The Trouble with Triples)
9. Going Places (vectors): see ME 104 (Vectors from Leibniz to Einstein)
10. Out of the Womb (matrices): see ME 98 (What is a Matrix?)
11. The Triumphs of Sisyphus (modular arithmetic): see ME 30 (The Roots of Unity) and 99 (The Triumphs of Sisyphus)
12. When Five Isn't Prime (ideals): see ME 101 (When Five Isn't Prime)
13. Marvelous Arithmetics of Distance (p -adic numbers): see ME 102 (Marvelous Arithmetics of Distance)
14. Numbers Far Afield (finite fields): see ME 66 (When $1+1$ Equals 0) and 103 (Numbers Far Afield)
15. Plus and Times Set Free (abstract algebra): see ME 106 (Plus and Times Set Free)