Studying Tilings At Radcliffe

What is it like to do mathematical research?

In coursework, one is usually presented with a polished theory together with problems that are known to have solutions. Research is messier, slower, and more exploratory. One spends a great deal of time looking at examples, noticing patterns, testing ideas, getting stuck, revising definitions, discovering that one's conjectures are false, and occasionally stumbling onto something new. That process can be frustrating, but it can also be exhilarating.

My own style of mathematical research is highly experimental and visual. I like to start with concrete examples and let the mathematics gradually reveal itself. Often the first sign that something mathematically significant is at work in a situation is a picture or an unexpected numerical pattern.

Computers are as essential to my work as telescopes are to astronomers. Even if the end result is a proof found by, and written for, human beings, my collaborators and I wouldn't have known what to try to prove if computer experiments hadn't revealed the phenomena at work.

I don't expect students to arrive already knowing how to do research. Learning how to explore the mathematical landscape is part of the project.

What kinds of students thrive in this setting?

Curiosity matters more than speed or prior knowledge. When I work with undergraduate research assistants, the ones who do best are often not the ones who answer questions fastest but the ones who tolerate uncertainty, enjoy playing with examples, are willing to experiment, and put in the time to think about a problem from various angles.

This project will place a strong emphasis on coding, at least in its initial phases. I'm looking for students who enjoy programming and want to use computation as part of mathematical exploration. Python experience would be especially helpful, and experience with SageMath would be even better.

Part of the work will involve building reusable computational infrastructure by translating and modernizing existing code, creating tools for generating and analyzing tilings and related combinatorial structures, producing images and animations, documenting software clearly, and designing code

that other researchers can adapt and extend. For example, one project will involve translating specialized C++ code into SageMath so that it can be integrated into a broader experimental framework.

Later stages of the project will involve using the tools we've created to find new mathematics. Some knowledge of combinatorics and discrete probability would be helpful, but the main prerequisite is enthusiasm for mathematics. To get a feeling for this subject, I suggest the Mathologer video "The Arctic Circle Theorem" (https://www.youtube.com/watch?v=Yy7Q8IWNfHM) and the essay I submitted to the Radcliffe Institute when I applied for a Fellowship (https://faculty.uml.edu/jpropp/Radcliffe.pdf).

Harvard already has many excellent opportunities for students whose mathematical development has been highly accelerated, but they're not the students I'm most eager to recruit; I'm more interested in finding students who are curious, creative, and collaborative but who don't yet see themselves as mathematical researchers.

What might you get out of doing this work?

Possible outcomes include learning how mathematical research actually works, developing stronger instincts for formulating conjectures and writing proofs, and gaining experience with mathematical computation and experimentation. There is also satisfaction in having contributed to mathematical knowledge. It is possible, though not guaranteed, that students' contributions will be significant enough to warrant coauthorship of a paper.

I expect students to invest 5 to 10 hours a week on the project, including time spent at hour-long weekly meetings and time spent writing up weekly progress reports. Students will have the opportunity to interact with Boston-area grad students, postdocs, and professors interested in tilings.

To get a sense of the possible outcomes of STAR, visit the website for a similar research group I ran at Harvard 25 years ago, called REACH:

https://faculty.uml.edu/jpropp/reach/

I try to create an environment in which students feel comfortable exploring unfamiliar ideas, making mistakes, following their curiosity, and gradually developing the confidence to contribute mathematically in their own way. Moments of joyful discovery cannot be forced, but they can be cultivated.