Freshman Seminar

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A few examples from courses most math majors take.

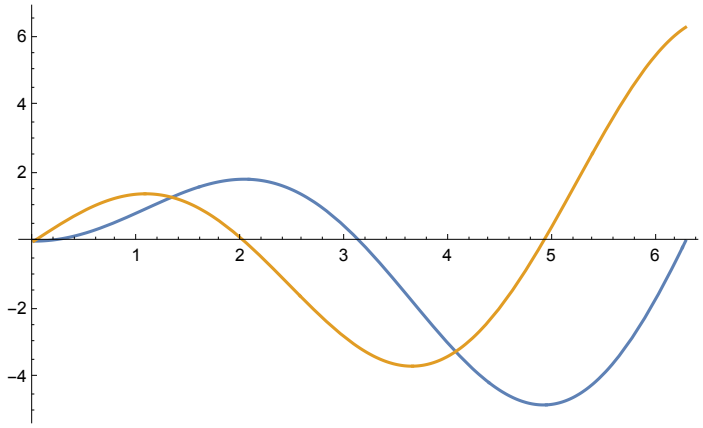
Calculus I/IA/IB

Differentiate and plot a function.

In[1]:=
```
f = Function[x, x Sin[x]]
```
Out[1]= $x \mapsto x \sin(x)$

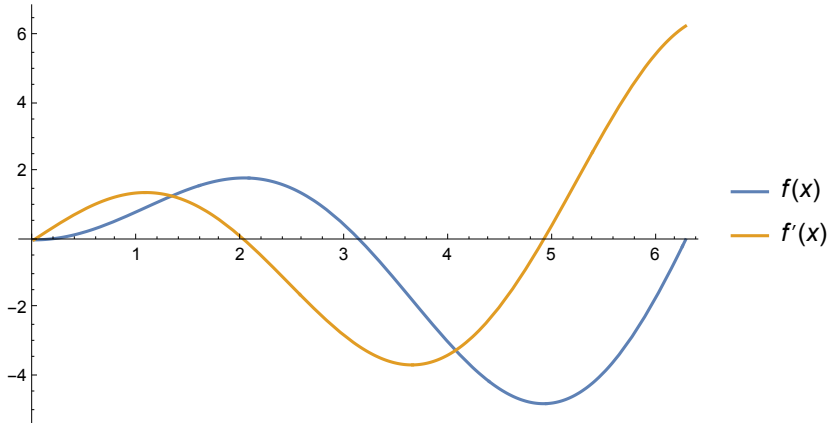
In[2]:=
```
f'
```
Out[2]= $x \mapsto \sin(x) + x \cos(x)$

In[3]:=
```
Plot[{f[x], f'[x]}, {x, 0, 2 Pi}]
```
Out[3]=



In[4]:=
```
Plot[{f[x], f'[x]}, {x, 0, 2 π}, PlotLegends → "Expressions"]
```

Out[4]=



Calculus II

Integrate an expression, indefinite and definite.

In[5]:=
```
y = x^2 E^(-x)
```

Out[5]= $e^{-x} x^2$

In[6]:=
```
∫y dx
```

Out[6]= $e^{-x} \left(-x^2 - 2x - 2\right)$

In[7]:=
```
Integrate[y, {x, 0, 5}]
```

Out[7]= $2 - \frac{37}{e^5}$

Evaluate an infinite series.

In[8]:=
```
Sum[(2 / 3)^n, {n, 0, ∞}]
```

Out[8]= 3

A Taylor series

In[9]:=
```
Series[Cos[x], {x, 0, 4}]
```

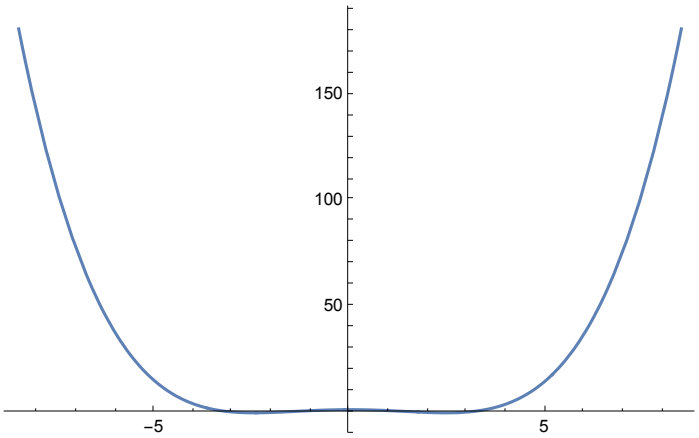
Out[9]= $1 - \frac{x^2}{2} + \frac{x^4}{24} + O\left(x^5\right)$

In[10]:=
```
Normal[1 - x^2/2 + x^4/24 + O[x]^5]
```

Out[10]= $\frac{x^4}{24} - \frac{x^2}{2} + 1$

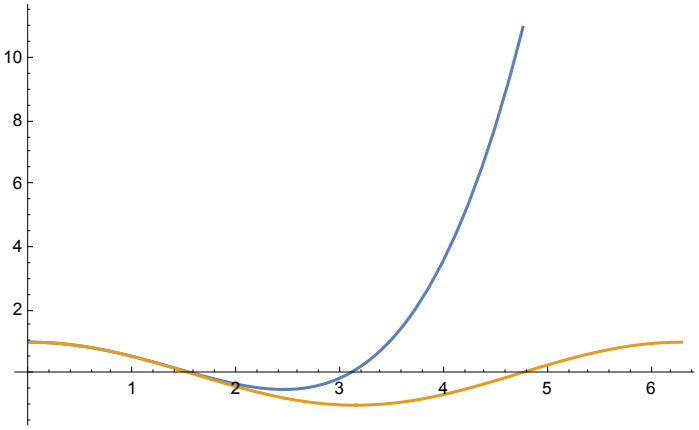
In[11]:=
```
Plot[1 - x^2/2 + x^4/24, {x, -8.48528, 8.48528}]
```

Out[11]=



In[12]:=
```
Plot[{1 - x^2/2 + x^4/24, Cos[x]}, {x, 0, 2 Pi}]
```

Out[12]=



Calculus III

In[13]:=
```
u = {4, -1, 1};
v = {1, 1, 2};
w = {2, -1, t};
```

In[16]:=
```
u.v
```

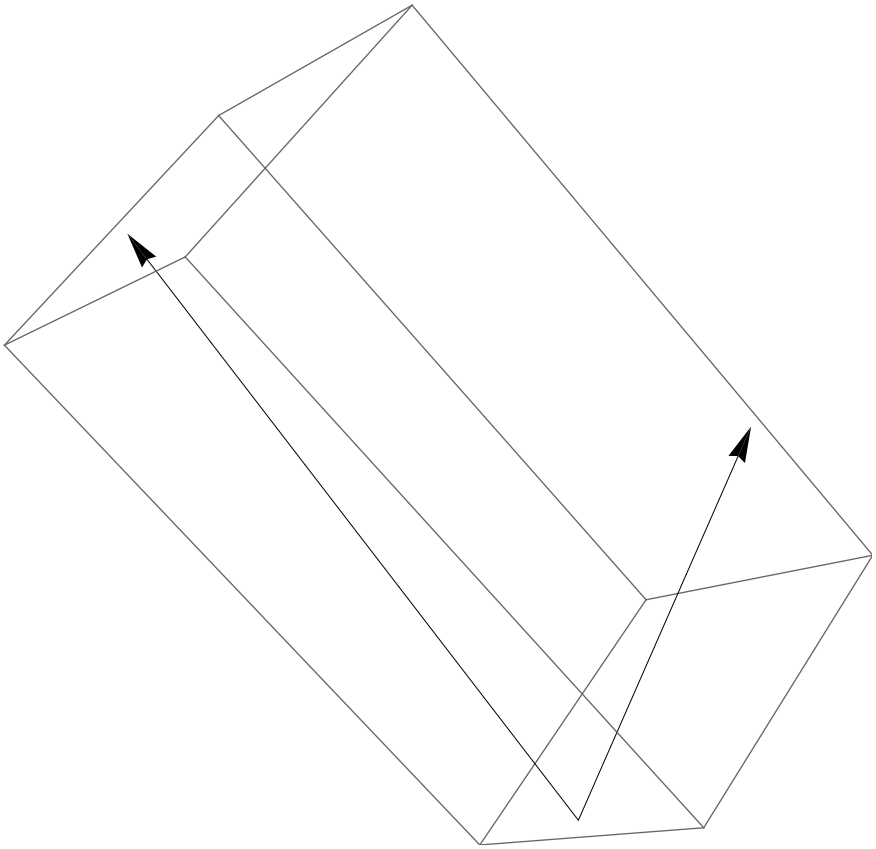
Out[16]= 5

In[18]:=
```
Solve[u.w == 0, t]
```

Out[18]= $\{\{t \to -9\}\}$

```
Graphics3D[{Arrow[{{0, 0, 0}, u}], Arrow[{{0, 0, 0}, v}]}]
```

Out[17]=



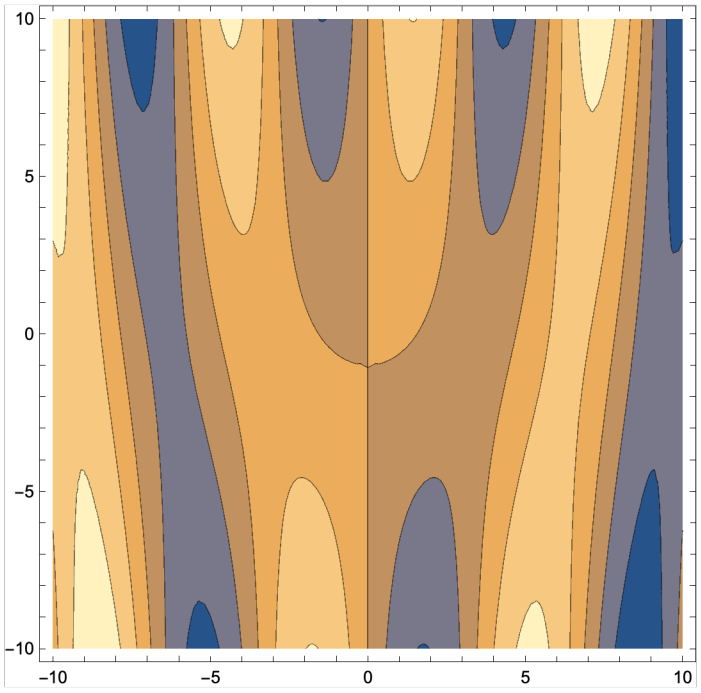
In[21]:=
```
Clear[x, y]
```

In[42]:=
```
z = x Cos[x] + y Sin[x]
```

Out[42]= $y \sin(x) + x \cos(x)$

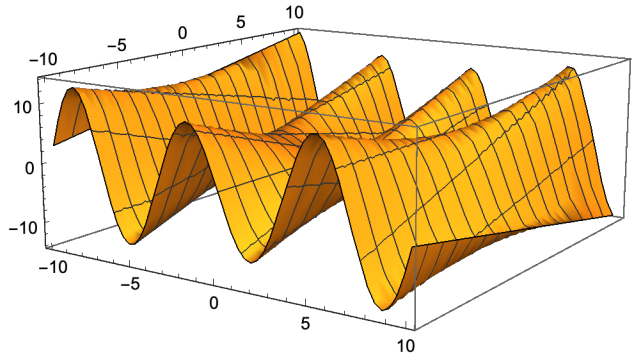
In[43]:=
```
ContourPlot[z, {x, -10, 10}, {y, -10, 10}]
```

Out[43]=



In[44]:=
```
Plot3D[z, {x, -10, 10}, {y, -10, 10}]
```

Out[44]=



Discrete Structures

Binary digits of a number

In[•]:=
```
IntegerDigits[1234567, 2]
```

Out[•]= {1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1}

The number of possible 5 card poker hands:

In[•]:=
```
Binomial[52, 5]
```

Out[•]= 2 598 960

A recursive definition

In[81]:=
```
F[0] = 0; F[1] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2]
```

In[86]:=
```
Table[F[i], {i, 0, 10}]
```

Out[86]= $\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$

Linear Algebra

Linear Equations

In[48]:=
```
Clear[x, y, z]
```

In[49]:=
```
Solve[{x + y + z == 4, x + 2 y - z == 5, x - y - z == 0}, {x, y, z}]
```

Out[49]= $\left\{\left\{x \to 2, y \to \frac{5}{3}, z \to \frac{1}{3}\right\}\right\}$

Matrix Equations

In[50]:=
```
A = {{2, -1, 0}, {-1, 3, -1}, {0, -1, 4}}
```

Out[50]= $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 4 \end{pmatrix}$

In[51]:=
```
Solve[A.{x, y, z} == {3, 4, 5}, {x, y, z}]
```

Out[51]= $\{\{x \to 3, y \to 3, z \to 2\}\}$

In[52]:=
```
Inverse[A]
```

Out[52]= $\begin{pmatrix} \frac{11}{18} & \frac{2}{9} & \frac{1}{18} \\ \frac{2}{9} & \frac{4}{9} & \frac{1}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{5}{18} \end{pmatrix}$

You can solve a matrix equation like the one above by multiplying both sides of the equation by the inverse matrix.

In[53]:=
```
Inverse[A].{3, 4, 5}
```

Out[53]= $\{3, 3, 2\}$

Differential Equations

In[54]:=
```
Clear[x, t]
```

In[55]:=
```
DSolve[x'[t] == -2 x[t] + t^2, x[t], t]
```

Out[55]= $\left\{\left\{x(t) \to c_1 e^{-2t} + \frac{1}{4}\left(2t^2 - 2t + 1\right)\right\}\right\}$

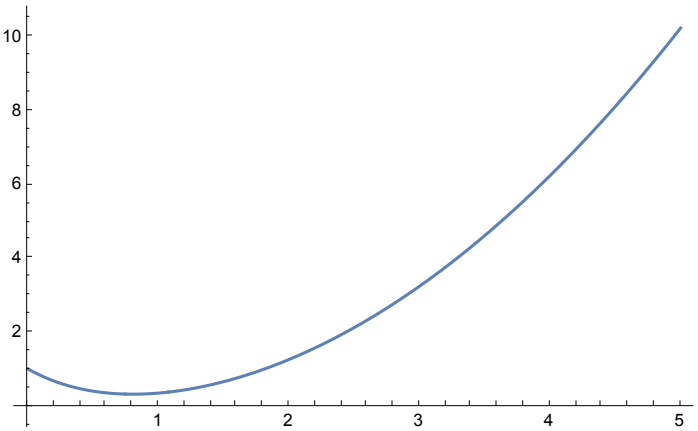
In[56]:=
```
DSolve[{x'[t] == -2 x[t] + t^2, x[0] == 1}, x[t], t]
```

Out[56]= $\left\{\left\{x(t) \to \frac{1}{4} e^{-2t}\left(2e^{2t}t^2 - 2e^{2t}t + e^{2t} + 3\right)\right\}\right\}$

In[57]:=
```
x[t_] := 1/4 e^(-2 t) (3 + e^(2 t) - 2 e^(2 t) t + 2 e^(2 t) t^2)
```

In[58]:=
```
Plot[x[t], {t, 0, 5}]
```

Out[58]=



Probability & Statistics

Distribution of probabilities for the roll of a fair die:

In[59]:=
```
roll = DiscreteUniformDistribution[{1, 6}]
```

Out[59]= DiscreteUniformDistribution[{1, 6}]

In[60]:=
```
RandomVariate[roll]
```

Out[60]= 3

In[61]:=
```
PDF[roll]
```

Out[61]= $\text{Function}\left[x, \begin{cases} \frac{1}{6} & 1 \le x \le 6 \\ 0 & \text{True} \end{cases}, \text{Listable}\right]$

In[72]:=
```
PDF[roll][{1, 2, 3}]
```

Out[72]= $\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$

In[62]:=
```
twodie = Total[RandomVariate[roll, 2]]
```

Out[62]= 9

In[66]:=
```
data = Table[Total[RandomVariate[roll, 2]], {1000}];
```

In[67]:=
```
Mean[data] // N
```

Out[67]= 6.909

In[68]:=
```
Histogram[data]
```

Out[68]=