Freshman Seminar - *Mathematica* Demo

Ken Levasseur (Kenneth_Levasseur@uml.edu)

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What is *Mathematica*?

Simple answer: *Mathematica* is a system for doing computation.

In[93]:=
```
$Version
```

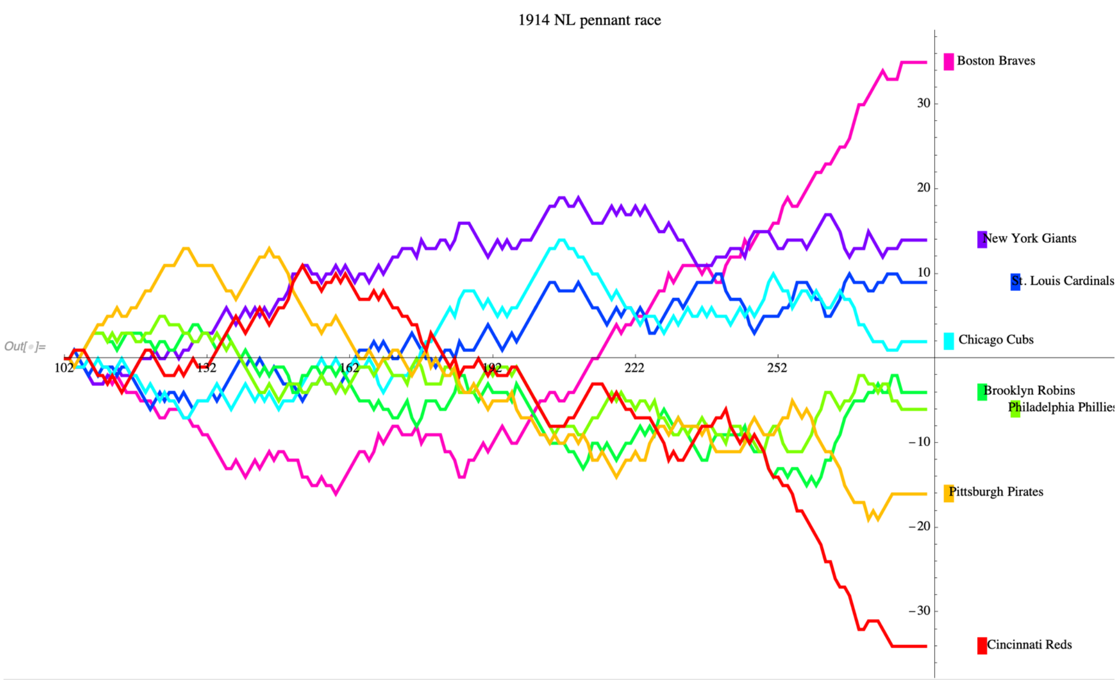
Out[93]=

13.1.0 for Mac OS X x86 (64-bit) (June 16, 2022)

Baseball

Out[ ]=

In[ ]:=
```
SeasonPlot[1914, "NL"]
```



How to Begin

- **Start typing : You get an input cell**

In[94]:=
```
Expand[(x + y) ^10]
```

Out[94]=

$$x^{10} + 10\,x^9\,y + 45\,x^8\,y^2 + 120\,x^7\,y^3 + 210\,x^6\,y^4 + 252\,x^5\,y^5 + 210\,x^4\,y^6 + 120\,x^3\,y^7 + 45\,x^2\,y^8 + 10\,x\,y^9 + y^{10}$$

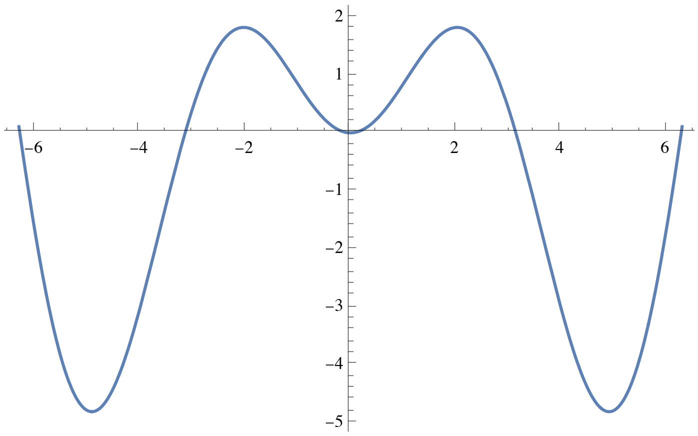
- **Starting with “=“ gives you a free-form/natural language input**

In[ ]:=

plot x sin x

```
Plot[x * Sin[x], {x, -6.3, 6.3}]
```

Out[95]=



- **Starting with “= =” gives you a Wolfram Alpha query**

In[96]:=

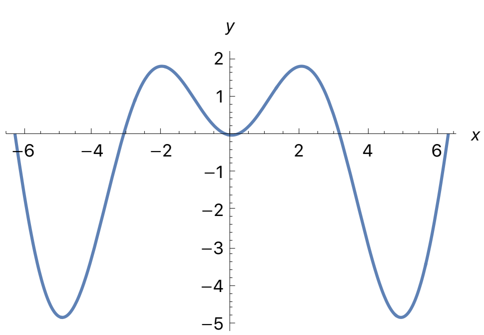
x sin x

Assuming multiplication | Use a list instead

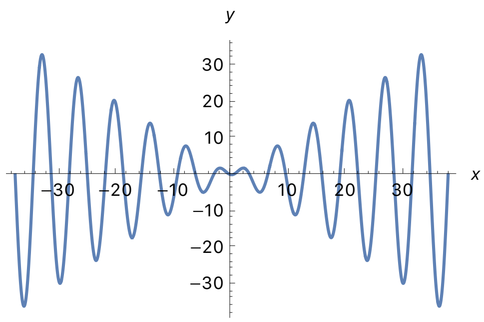
Input:

$x \sin(x)$

Plots:



min max



min max

Alternate form:

$$\frac{1}{2} i e^{-i x} x - \frac{1}{2} i e^{i x} x$$

Roots:

Approximate form

$x = \pi n, \quad n \in \mathbb{Z}$

Z is the set of integers »

Integer root:

$x = 0$

Properties as a real function:

Domain:

$\mathbb{R}$ (all real numbers)

Range:

$\mathbb{R}$ (all real numbers)

Surjectivity:

surjective onto $\mathbb{R}$

Parity:

even

R is the set of real numbers »

Series expansion at x = 0:

$$x^2 - \frac{x^4}{6} + \frac{x^6}{120} + O(x^7)$$

(Taylor series)

Big-O notation »

Derivative:

Step-by-step solution

$$\frac{d}{dx}(x \sin(x)) = \sin(x) + x \cos(x)$$

Indefinite integral: Step-by-step solution

$$\int x \sin(x)\, dx = \sin(x) - x\cos(x) + \text{constant}$$

Local maxima: More

$$\max\{x \sin(x)\} \approx 1.81971 \text{ at } x \approx -2.02876$$

$$\max\{x \sin(x)\} \approx 1.81971 \text{ at } x \approx 2.02876$$

Local minima: More

$$\min\{x \sin(x)\} = 0 \text{ at } x = 0$$

$$\min\{x \sin(x)\} \approx -4.81447 \text{ at } x \approx -4.91318$$

Alternative representations: More

$$x \sin(x) = \frac{x}{\csc(x)}$$

$$x \sin(x) = -x \cos\left(\frac{\pi}{2} + x\right)$$

$$x \sin(x) = x \cos\left(\frac{\pi}{2} - x\right)$$

csc(x) is the cosecant function »

More information »

Series representations: More

$$x \sin(x) = x \sum_{k=0}^{\infty} \frac{(-1)^k x^{1+2k}}{(1+2k)!}$$

$$x \sin(x) \propto \frac{x \sum_{k=0}^{\infty} (-1)^k \frac{\partial^{2k} \delta(x)}{\partial x^{2k}}}{\theta(x)}$$

$$x \sin(x) = 2x \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(x)$$

n! is the factorial function »

$\theta(x)$ is the Heaviside step function »

$\delta(x)$ is the Dirac delta function »

J_n(z) is the Bessel function of the first kind »

More information »

Integral representations:

$$x \sin(x) = x^2 \int_0^1 \cos(t x)\, dt$$

$$x \sin(x) = -\frac{i x^2}{4\sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{s - x^2/(4s)}}{s^{3/2}}\, ds \text{ for } \gamma > 0$$

$$x\sin(x) = -\frac{i\,x}{2\sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-1+2s}\,x^{1-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } (0<\gamma<1 \text{ and } x>0)$$

$\Gamma(x)$ is the gamma function »

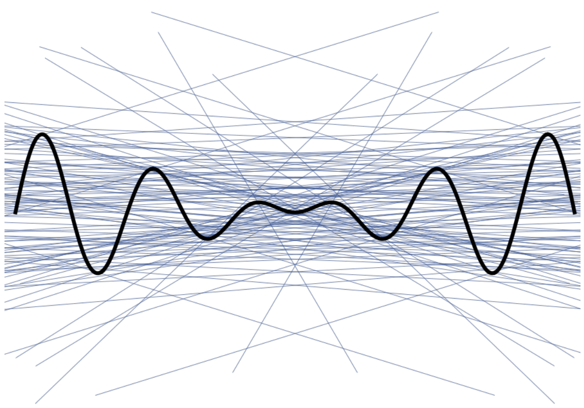
More information »

Definite integral:

More digits

$$\int_0^\pi x\sin(x)\,dx = \pi \approx 3.14159$$

Differential geometric curves:



— $x\sin(x)$ — normals

Horizontal plot range:

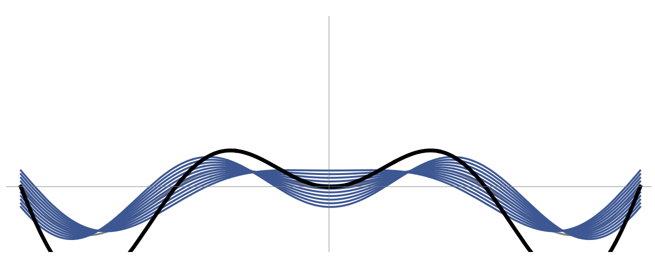
x_{min} x_{max} symmetric

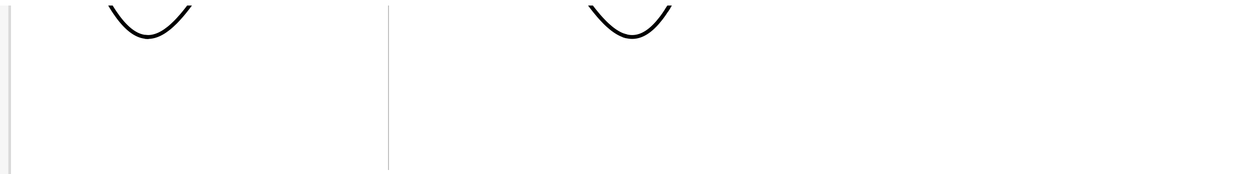
+ More controls

Differential equation solution curve families:

embedding function $c_3\sin(x) + c_4\,x\sin(x) + c_1\cos(x) + c_2\,x\cos(x)$

embedding ODE $y^{(4)}(x) + 2\,y''(x) + y(x) = 0$





— original function $x \sin(x)$

— c_j-values function $\frac{1}{2} x \sin(x)$

— c_1-family $c_1 \cos(x) + \frac{1}{2} x \sin(x)$

Horizontal plot range

x_{min} x_{max}

show axes ✓ show ticks

+ More controls

WolframAlpha

A few examples from courses most math majors take.

■ Precalculus/Trig

In[97]:=
```
Solve[2 x^2 + 5 x - 7 == 0, x]
```

Out[97]=

$$\left\{\left\{x \to -\frac{7}{2}\right\}, \{x \to 1\}\right\}$$

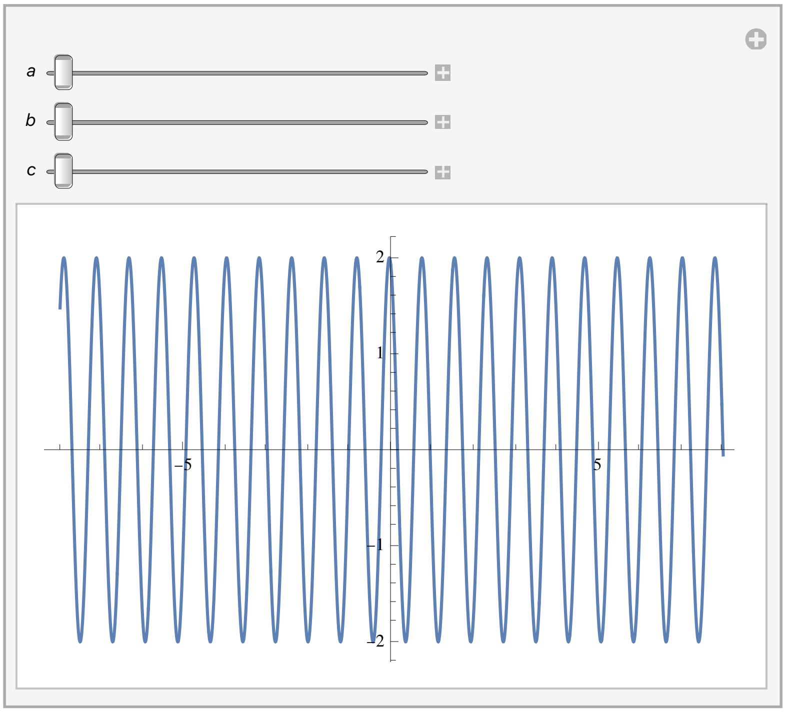
In[98]:=
```
c Sin[a x + b]
```

Out[98]=

$$c \sin(a x + b)$$

In[99]:=
```
Manipulate[Plot[c Sin[b + a x], {x, -8, 8}], {a, -8, 8}, {b, -2, 2}, {c, -2, 2}]
```

Out[99]=



In[100]:=
```
Solve[x Sin[x] == 1 / 5, x]
```

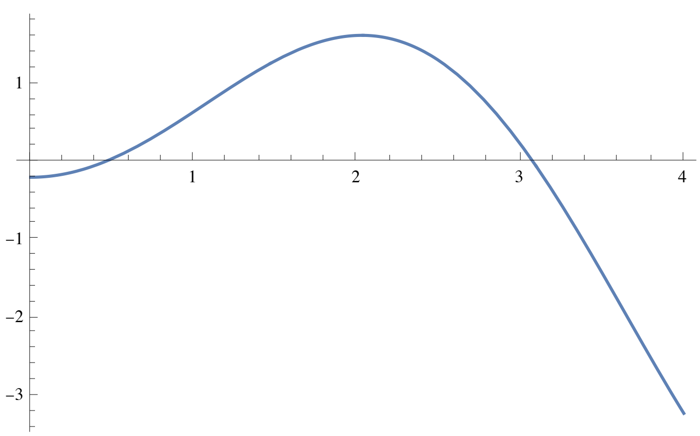
Solve: This system cannot be solved with the methods available to Solve.

Out[100]=

$$\text{Solve}\left[x \sin(x) = \frac{1}{5}, x\right]$$

In[101]:=
```
Plot[x Sin[x] - 1 / 5, {x, 0, 4}]
```

Out[101]=



In[102]:=
```
FindRoot[x Sin[x] == 1 / 5, {x, 0.5}]
```

Out[102]=

$\{x \to 0.455053\}$

In[103]:=

```
FindRoot[x Sin[x] == 1 / 5, {x, 3}]
```

Out[103]=

$\{x \to 3.07654\}$

▪ Calculus I/IA/IB

Differentiate and plot a function.

In[104]:=

```
f = Function[x, x Sin[x]]
```

Out[104]=

$x \mapsto x \sin(x)$

In[105]:=

```
f′
```

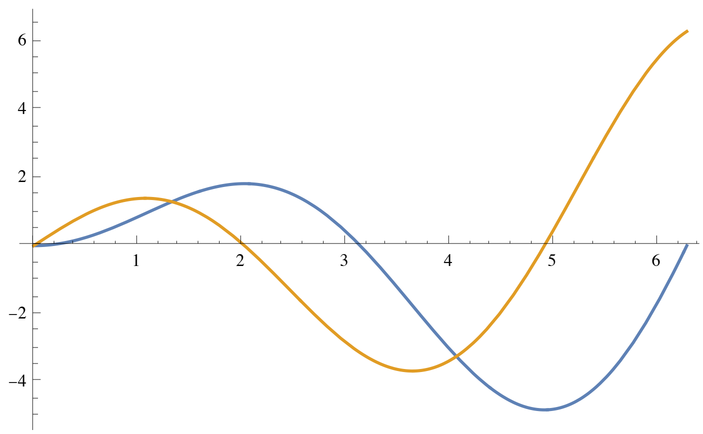
Out[105]=

$x \mapsto \sin(x) + x \cos(x)$

In[106]:=

```
Plot[{Tooltip[f[x]], Tooltip[f'[x]]}, {x, 0, 2 Pi}]
```

Out[106]=



▪ Calculus II

Integrate an expression, indefinite and definite.

In[107]:=

```
y = x ^ 2 E ^ (- x)
```

Out[107]=

$e^{-x} x^2$

In[108]:=

```
Integrate[y, x]
```

Out[108]=

$e^{-x} \left(-x^2 - 2 x - 2\right)$

In[109]:=

```
Integrate[y, {x, 0, 5}]
```

Out[109]=

$$2 - \frac{37}{e^5}$$

Evaluate an infinite series.

In[110]:=

```
Sum[(2 / 3) ^ n, {n, 0, ∞}]
```

Out[110]=

3

A Taylor series

In[111]:=

```
Series[Cos[x], {x, 0, 4}]
```

Out[111]=

$$1 - \frac{x^2}{2} + \frac{x^4}{24} + O(x^5)$$

■ Calculus III

In[112]:=

```
u = {4, -2, 1};
v = {1, 1, -2};
```

In[114]:=

```
u.v
```

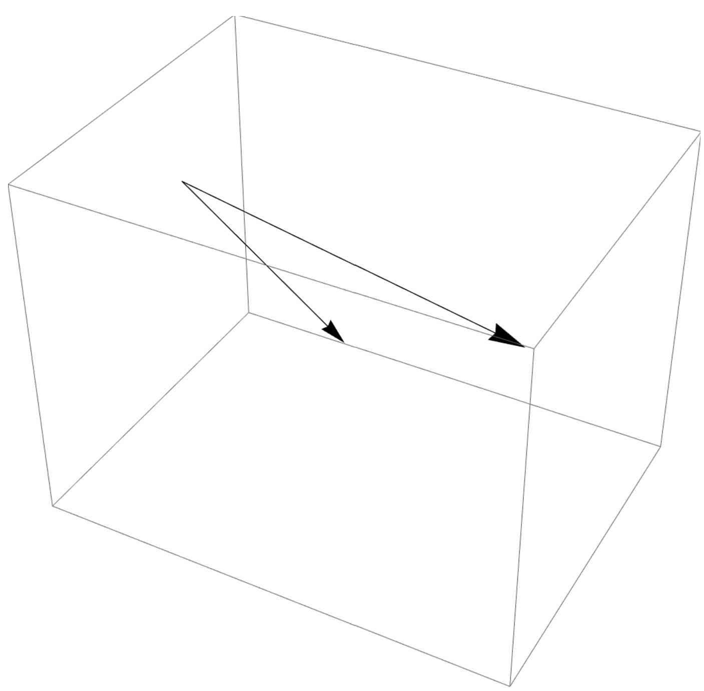
Out[114]=

0

In[115]:=

```
Graphics3D[{Arrow[{{0, 0, 0}, u}], Arrow[{{0, 0, 0}, v}]}]
```

Out[115]=



In[116]:=

```
Clear[x, y]
```

In[117]:=

```
z = x ^ 2 - y ^ 2
```

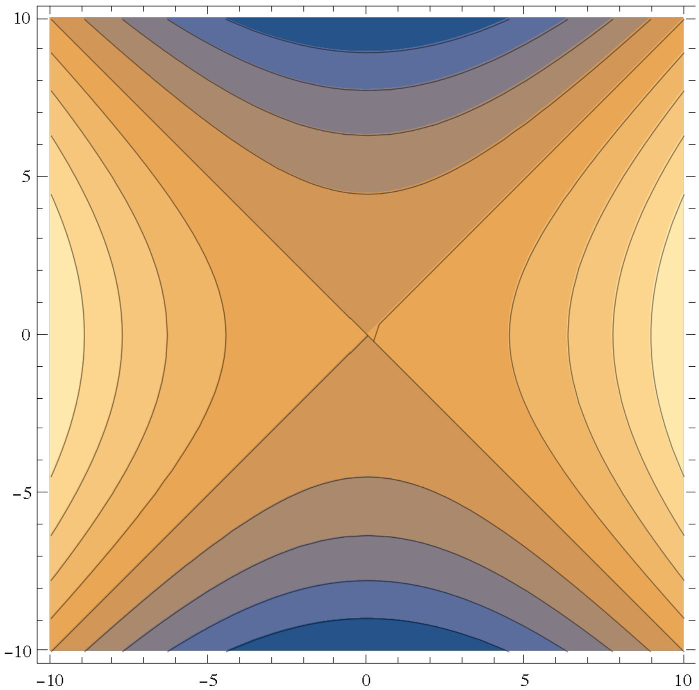
Out[117]=

$x^2 - y^2$

In[118]:=

```
ContourPlot[z, {x, -10, 10}, {y, -10, 10}]
```

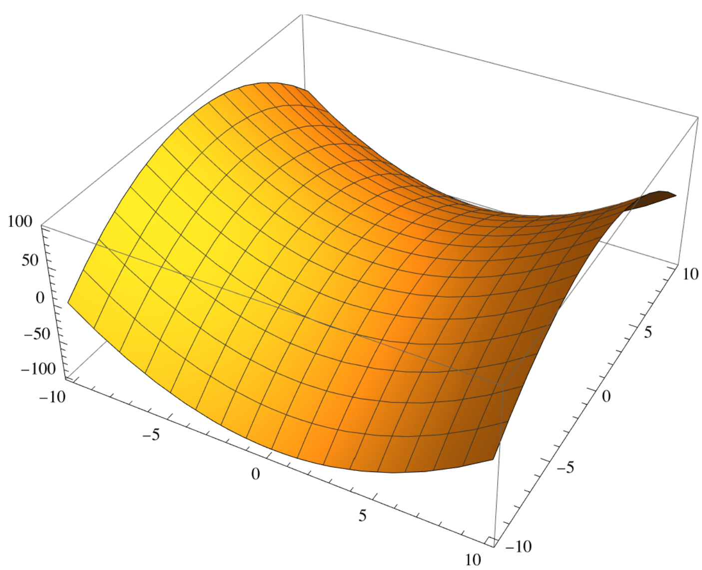
Out[118]=



In[119]:=

```
Plot3D[z, {x, -10, 10}, {y, -10, 10}]
```

Out[119]=



■ Discrete Structures

Binary digits of a number

In[120]:=

```
IntegerDigits[1 234 567, 2]
```

Out[120]=

{1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1}

The number of possible 5 card poker hands:

In[121]:=

```
Binomial[52, 5]
```

Out[121]=

2 598 960

A recursive definition

In[122]:=

```
Clear[f]
```

In[123]:=

```
f[0] = 0; f[1] = 1;
f[n_] := f[n] = f[n - 1] + f[n - 2]
```

In[125]:=

```
Table[f[i], {i, 0, 10}]
```

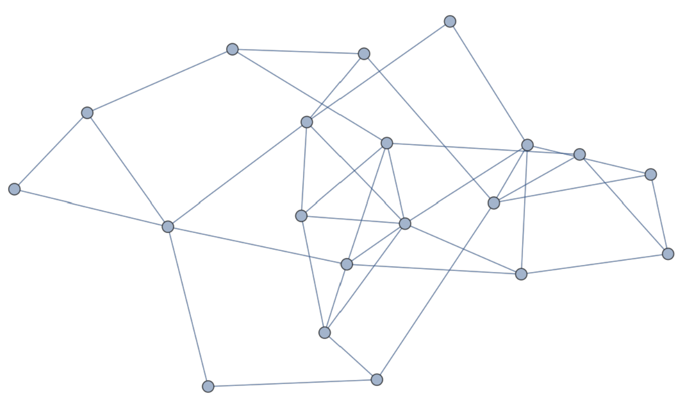
Out[125]=

{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55}

In[126]:=

```
RandomGraph[BernoulliGraphDistribution[20, 1/5]]
```

Out[126]=



▪ Linear Algebra

Linear Equations

In[127]:=

```
Clear[x, y, z]
```

In[128]:=

```
Solve[{x + y + z == 4, x + 2 y - z == 5, x - y - z == 0}, {x, y, z}]
```

Out[128]=

$$\left\{\left\{x \to 2,\ y \to \frac{5}{3},\ z \to \frac{1}{3}\right\}\right\}$$

Matrix Equations

In[129]:=

```
A = ( 2  -1  0
     -1   3 -1
      0  -1  4 )
```

Out[129]=

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 4 \end{pmatrix}$$

In[130]:=
```
Solve[A.{x, y, z} == {3, 4, 5}, {x, y, z}]
```
Out[130]=
$\{\{x \to 3, y \to 3, z \to 2\}\}$

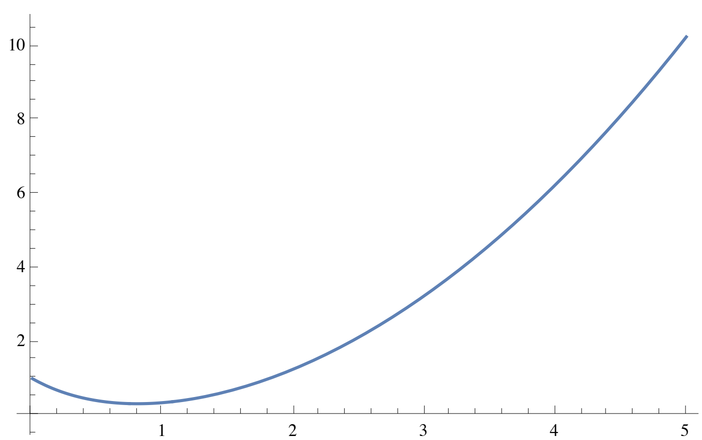
In[131]:=
```
Inverse[A]
```
Out[131]=
$$\begin{pmatrix} \frac{11}{18} & \frac{2}{9} & \frac{1}{18} \\ \frac{2}{9} & \frac{4}{9} & \frac{1}{9} \\ \frac{1}{18} & \frac{1}{9} & \frac{5}{18} \end{pmatrix}$$

■ Differential Equations

In[132]:=
```
Clear[x, t]
```
In[133]:=
```
DSolve[x'[t] == -2 x[t] + t^2, x[t], t]
```
Out[133]=
$$\left\{\left\{x(t) \to \frac{1}{4}\left(2 t^2 - 2 t + 1\right) + c_1 e^{-2 t}\right\}\right\}$$

In[134]:=
```
DSolve[{x'[t] == -2 x[t] + t^2, x[0] == 1}, x[t], t]
```
Out[134]=
$$\left\{\left\{x(t) \to \frac{1}{4} e^{-2 t}\left(2 e^{2 t} t^2 - 2 e^{2 t} t + e^{2 t} + 3\right)\right\}\right\}$$

In[135]:=
$$x[t\_] := \frac{1}{4} e^{-2 t}\left(3 + e^{2 t} - 2 e^{2 t} t + 2 e^{2 t} t^2\right)$$

In[136]:=
```
Plot[x[t], {t, 0, 5}]
```
Out[136]=


■ Probability & Statistics

Distribution of probabilities for the roll of a fair die:

In[137]:=
```
roll = DiscreteUniformDistribution[{1, 6}]
```
Out[137]=
DiscreteUniformDistribution[{1, 6}]

In[138]:=
```
RandomVariate[roll]
```
Out[138]=
6

In[139]:=
```
PDF[roll]
```
Out[139]=
$$\text{Function}\left[x, \begin{cases} \frac{1}{6} & 1 \le x \le 6 \\ 0 & \text{True} \end{cases}, \text{Listable}\right]$$

In[140]:=
```
twodice := Total[RandomVariate[roll, 2]]
```
In[141]:=
```
{twodice, twodice, twodice}
```
Out[141]=
{6, 8, 5}

In[142]:=
```
data = Table[twodice, {10 000}];
```
In[143]:=
```
Mean[data] // N
```
Out[143]=
7.0182

In[144]:=
```
Histogram[data]
```
Out[144]=