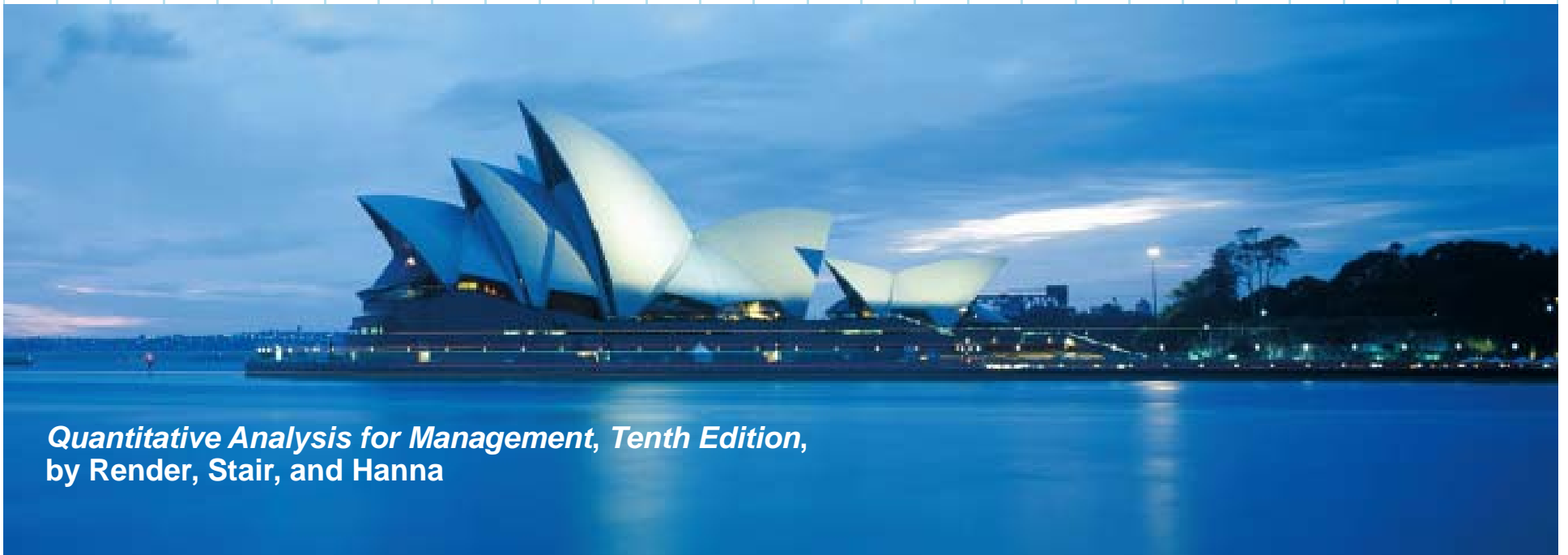


# ***Chapter 3***

# ***Decision Analysis***



*Quantitative Analysis for Management, Tenth Edition,*  
by Render, Stair, and Hanna

# *Learning Objectives*

After completing this chapter, students will be able to:

- 1. List the steps of the decision-making process**
- 2. Describe the types of decision-making environments**
- 3. Make decisions under uncertainty**
- 4. Use probability values to make decisions under risk**

# *Learning Objectives*

After completing this chapter, students will be able to:

- 5. Develop accurate and useful decision trees**
- 6. Revise probabilities using Bayesian analysis**
- 7. Use computers to solve basic decision-making problems**
- 8. Understand the importance and use of utility theory in decision making**

# ***Chapter Outline***

- 3.1 Introduction**
- 3.2 The Six Steps in Decision Making**
- 3.3 Types of Decision-Making Environments**
- 3.4 Decision Making under Uncertainty**
- 3.5 Decision Making under Risk**
- 3.6 Decision Trees**
- 3.7 How Probability Values Are Estimated by Bayesian Analysis**
- 3.8 Utility Theory**

# ***Introduction***

- **What is involved in making a good decision?**
- **Decision theory is an analytic and systematic approach to the study of decision making**
- **A good decision is one that is based on logic, considers all available data and possible alternatives, and the quantitative approach described here**

# ***The Six Steps in Decision Making***

- 1. Clearly define the problem at hand**
- 2. List the possible alternatives**
- 3. Identify the possible outcomes or states of nature**
- 4. List the payoff or profit of each combination of alternatives and outcomes**
- 5. Select one of the mathematical decision theory models**
- 6. Apply the model and make your decision**

# *Thompson Lumber Company*

## **Step 1 – Define the problem**

- **Expand by manufacturing and marketing a new product, backyard storage sheds**

## **Step 2 – List alternatives**

- **Construct a large new plant**
- **A small plant**
- **No plant at all**

## **Step 3 – Identify possible outcomes**

- **The market could be favorable or unfavorable**

# *Thompson Lumber Company*

## **Step 4 – List the payoffs**

- Identify *conditional values* for the profits for large, small, and no plants for the two possible market conditions

## **Step 5 – Select the decision model**

- Depends on the environment and amount of risk and uncertainty

## **Step 6 – Apply the model to the data**

- Solution and analysis used to help the decision making

# *Thompson Lumber Company*

ALTERNATIVE	STATE OF NATURE	
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)
Construct a large plant	200,000	-180,000
Construct a small plant	100,000	-20,000
Do nothing	0	0

Table 3.1

# *Types of Decision-Making Environments*

## **Type 1: Decision making under certainty**

- Decision maker *knows with certainty* the consequences of every alternative or decision choice

## **Type 2: Decision making under uncertainty**

- The decision maker *does not know* the probabilities of the various outcomes

## **Type 3: Decision making under risk**

- The decision maker *knows the probabilities* of the various outcomes

# ***Decision Making Under Uncertainty***

**There are several criteria for making decisions under uncertainty**

- 1. Maximax (optimistic)**
- 2. Maximin (pessimistic)**
- 3. Criterion of realism (Hurwicz)**
- 4. Equally likely (Laplace)**
- 5. Minimax regret**

# Maximax

Used to find the alternative that maximizes the maximum payoff

- Locate the maximum payoff for each alternative
- Select the alternative with the maximum number

ALTERNATIVE	STATE OF NATURE		MAXIMUM IN A ROW (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	200,000
Construct a small plant	100,000	-20,000	100,000
Do nothing	0	0	0

200,000  
Maximax

Table 3.2

# Maximin

Used to find the alternative that maximizes the minimum payoff

- Locate the minimum payoff for each alternative
- Select the alternative with the maximum number

ALTERNATIVE	STATE OF NATURE		MINIMUM IN A ROW (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	-180,000
Construct a small plant	100,000	-20,000	-20,000
Do nothing	0	0	0

**Maximin**

Table 3.3

# ***Criterion of Realism (Hurwicz)***

A ***weighted average*** compromise between optimistic and pessimistic

- Select a coefficient of realism  $\alpha$
- Coefficient is between 0 and 1
- A value of 1 is 100% optimistic
- Compute the weighted averages for each alternative
- Select the alternative with the highest value

$$\text{Weighted average} = \alpha(\text{maximum in row}) + (1 - \alpha)(\text{minimum in row})$$

# Criterion of Realism (Hurwicz)

- For the large plant alternative using  $\alpha = 0.8$   
 $(0.8)(200,000) + (1 - 0.8)(-180,000) = 124,000$
- For the small plant alternative using  $\alpha = 0.8$   
 $(0.8)(100,000) + (1 - 0.8)(-20,000) = 76,000$

ALTERNATIVE	STATE OF NATURE		CRITERION OF REALISM ( $\alpha = 0.8$ )\$
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	124,000 Realism
Construct a small plant	100,000	-20,000	76,000
Do nothing	0	0	0

Table 3.4

# *Equally Likely (Laplace)*

Considers all the payoffs for each alternative

- Find the average payoff for each alternative
- Select the alternative with the highest average

ALTERNATIVE	STATE OF NATURE		ROW AVERAGE (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	10,000
Construct a small plant	100,000	-20,000	40,000
Do nothing	0	0	0

Equally likely

Table 3.5

# *Minimax Regret*

Based on *opportunity loss* or *regret*, the difference between the optimal profit and actual payoff for a decision

- Create an opportunity loss table by determining the opportunity loss for not choosing the best alternative
- Opportunity loss is calculated by subtracting each payoff in the column from the best payoff in the column
- Find the maximum opportunity loss for each alternative and pick the alternative with the minimum number

# Minimax Regret

■ Opportunity Loss Tables

STATE OF NATURE	
FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)
200,000 – 200,000	0 – (-180,000)
200,000 – 100,000	0 – (-20,000)
200,000 – 0	0 – 0

Table 3.6

ALTERNATIVE	STATE OF NATURE	
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)
Construct a large plant	0	180,000
Construct a small plant	100,000	20,000
Do nothing	200,000	0

Table 3.7

# Minimax Regret

ALTERNATIVE	STATE OF NATURE		MAXIMUM IN A ROW (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	0	180,000	180,000
Construct a small plant	100,000	20,000	100,000
Do nothing	200,000	0	200,000

100,000  
Minimax  
200,000

Table 3.8

# *Decision Making Under Risk*

- Decision making when there are several possible states of nature and we know the probabilities associated with each possible state
- Most popular method is to choose the alternative with the highest *expected monetary value (EMV)*

EMV (alternative  $i$ ) = (payoff of first state of nature)  
x (probability of first state of nature)  
+ (payoff of second state of nature)  
x (probability of second state of nature)  
+ ... + (payoff of last state of nature)  
x (probability of last state of nature)

# *EMV for Thompson Lumber*

- Each market has a probability of 0.50
- Which alternative would give the highest EMV?
- The calculations are

$$\begin{aligned}\text{EMV (large plant)} &= (0.50)(\$200,000) + (0.50)(-\$180,000) \\ &= \$10,000\end{aligned}$$

$$\begin{aligned}\text{EMV (small plant)} &= (0.50)(\$100,000) + (0.50)(-\$20,000) \\ &= \$40,000\end{aligned}$$

$$\begin{aligned}\text{EMV (do nothing)} &= (0.50)(\$0) + (0.50)(\$0) \\ &= \$0\end{aligned}$$

# *EMV for Thompson Lumber*

ALTERNATIVE	STATE OF NATURE		EMV (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	10,000
Construct a small plant	100,000	-20,000	40,000
Do nothing	0	0	0
Probabilities	0.50	0.50	

Table 3.9

**Largest EMV**



# ***Expected Value of Perfect Information (EVPI)***

- **EVPI places an upper bound on what you should pay for additional information**

$$\text{EVPI} = \text{EV}_{\text{wPI}} - \text{Maximum EMV}$$

- **EV<sub>wPI</sub> is the long run average return if we have perfect information before a decision is made**

$$\begin{aligned} \text{EV}_{\text{wPI}} = & \text{(best payoff for first state of nature)} \\ & \times \text{(probability of first state of nature)} \\ & + \text{(best payoff for second state of nature)} \\ & \times \text{(probability of second state of nature)} \\ & + \dots + \text{(best payoff for last state of nature)} \\ & \times \text{(probability of last state of nature)} \end{aligned}$$

## ***Expected Value of Perfect Information (EVPI)***

- **Scientific Marketing, Inc. offers analysis that will provide certainty about market conditions (favorable)**
- **Additional information will cost \$65,000**
- **Is it worth purchasing the information?**

## ***Expected Value of Perfect Information (EVPI)***

- 1. Best alternative for favorable state of nature is build a large plant with a payoff of \$200,000  
Best alternative for unfavorable state of nature is to do nothing with a payoff of \$0**

$$\text{EV}_{\text{wPI}} = (\$200,000)(0.50) + (\$0)(0.50) = \$100,000$$

- 2. The maximum EMV without additional information is \$40,000**

$$\begin{aligned}\text{EVPI} &= \text{EV}_{\text{wPI}} - \text{Maximum EMV} \\ &= \$100,000 - \$40,000 \\ &= \$60,000\end{aligned}$$

# *Expected Value of Perfect Information (EVPI)*

1. Best alternative is to build a large plant  
Best alternative is to do nothing  
EV<sub>w</sub>PI =

So the maximum Thompson should pay for the additional information is \$60,000

2. The maximum EMV without additional information is \$40,000

$$\begin{aligned} \text{EVPI} &= \text{EV}_{\text{wPI}} - \text{Maximum EMV} \\ &= \$100,000 - \$40,000 \\ &= \$60,000 \end{aligned}$$

# *Expected Opportunity Loss*

- *Expected opportunity loss* (EOL) is the cost of not picking the best solution
- First construct an opportunity loss table
- For each alternative, multiply the opportunity loss by the probability of that loss for each possible outcome and add these together
- Minimum EOL will always result in the same decision as maximum EMV
- Minimum EOL will always equal EVPI

# Expected Opportunity Loss

ALTERNATIVE	STATE OF NATURE		EOL
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	0	180,000	90,000
Construct a small plant	100,000	20,000	60,000
Do nothing	200,000	0	100,000
Probabilities	0.50	0.50	

Table 3.10

**Minimum EOL**

$$\begin{aligned} \text{EOL (large plant)} &= (0.50)(\$0) + (0.50)(\$180,000) \\ &= \$90,000 \end{aligned}$$

$$\begin{aligned} \text{EOL (small plant)} &= (0.50)(\$100,000) + (0.50)(\$20,000) \\ &= \$60,000 \end{aligned}$$

$$\begin{aligned} \text{EOL (do nothing)} &= (0.50)(\$200,000) + (0.50)(\$0) \\ &= \$100,000 \end{aligned}$$

# ***Sensitivity Analysis***

- **Sensitivity analysis examines how our decision might change with different input data**
- **For the Thompson Lumber example**

**$P$  = probability of a favorable market**

**$(1 - P)$  = probability of an unfavorable market**

# ***Sensitivity Analysis***

$$\begin{aligned}\text{EMV(Large Plant)} &= \$200,000P - \$180,000(1 - P) \\ &= \$200,000P - \$180,000 + \$180,000P \\ &= \$380,000P - \$180,000\end{aligned}$$

$$\begin{aligned}\text{EMV(Small Plant)} &= \$100,000P - \$20,000(1 - P) \\ &= \$100,000P - \$20,000 + \$20,000P \\ &= \$120,000P - \$20,000\end{aligned}$$

$$\begin{aligned}\text{EMV(Do Nothing)} &= \$0P + 0(1 - P) \\ &= \$0\end{aligned}$$

# Sensitivity Analysis

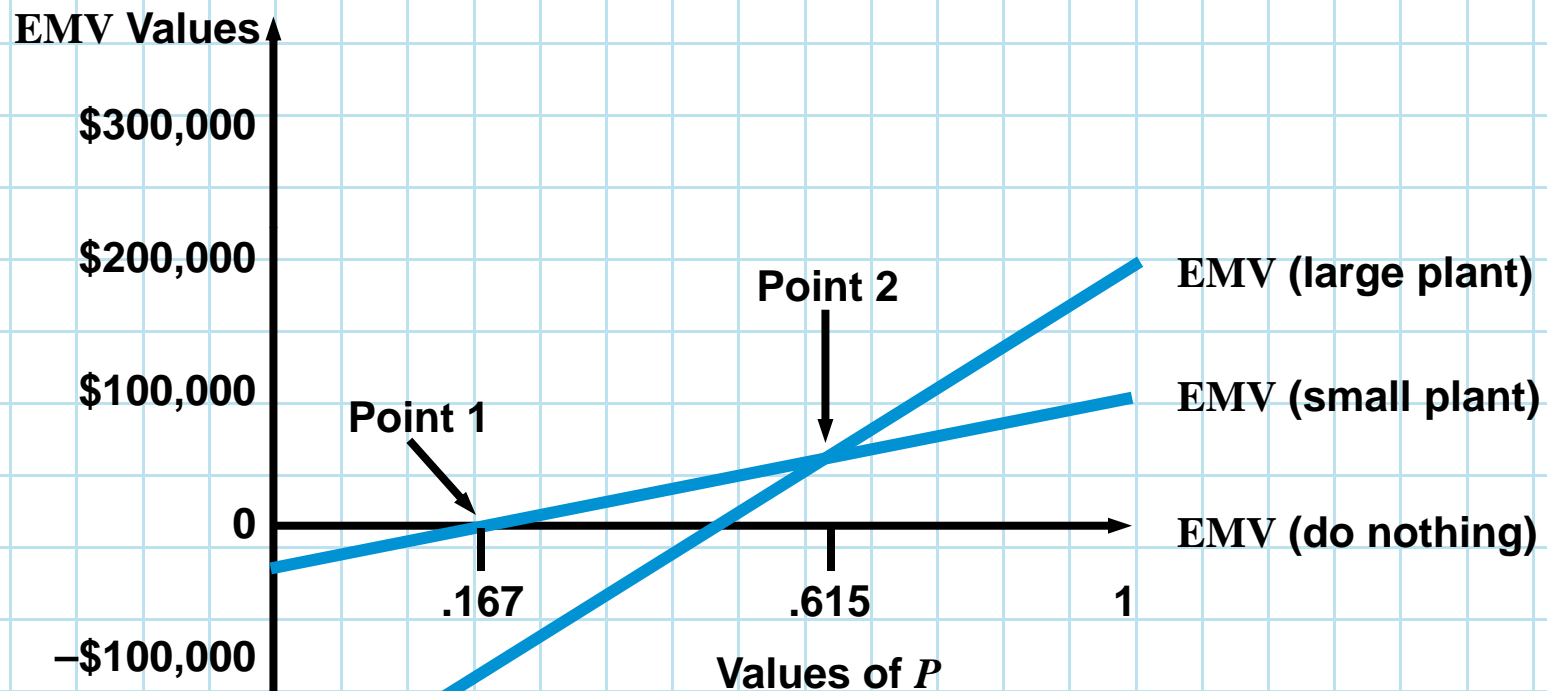


Figure 3.1

# *Sensitivity Analysis*

## **Point 1:**

**EMV(do nothing) = EMV(small plant)**

$$0 = \$120,000P - \$20,000 \quad P = \frac{20,000}{120,000} = 0.167$$

## **Point 2:**

**EMV(small plant) = EMV(large plant)**

$$\$120,000P - \$20,000 = \$380,000P - \$180,000$$

$$P = \frac{160,000}{260,000} = 0.615$$

# Sensitivity Analysis

**BEST  
ALTERNATIVE**

**RANGE OF  $P$   
VALUES**

**Do nothing**

**Less than 0.167**

**Construct a small plant**

**0.167 – 0.615**

**Construct a large plant**

**Greater than 0.615**

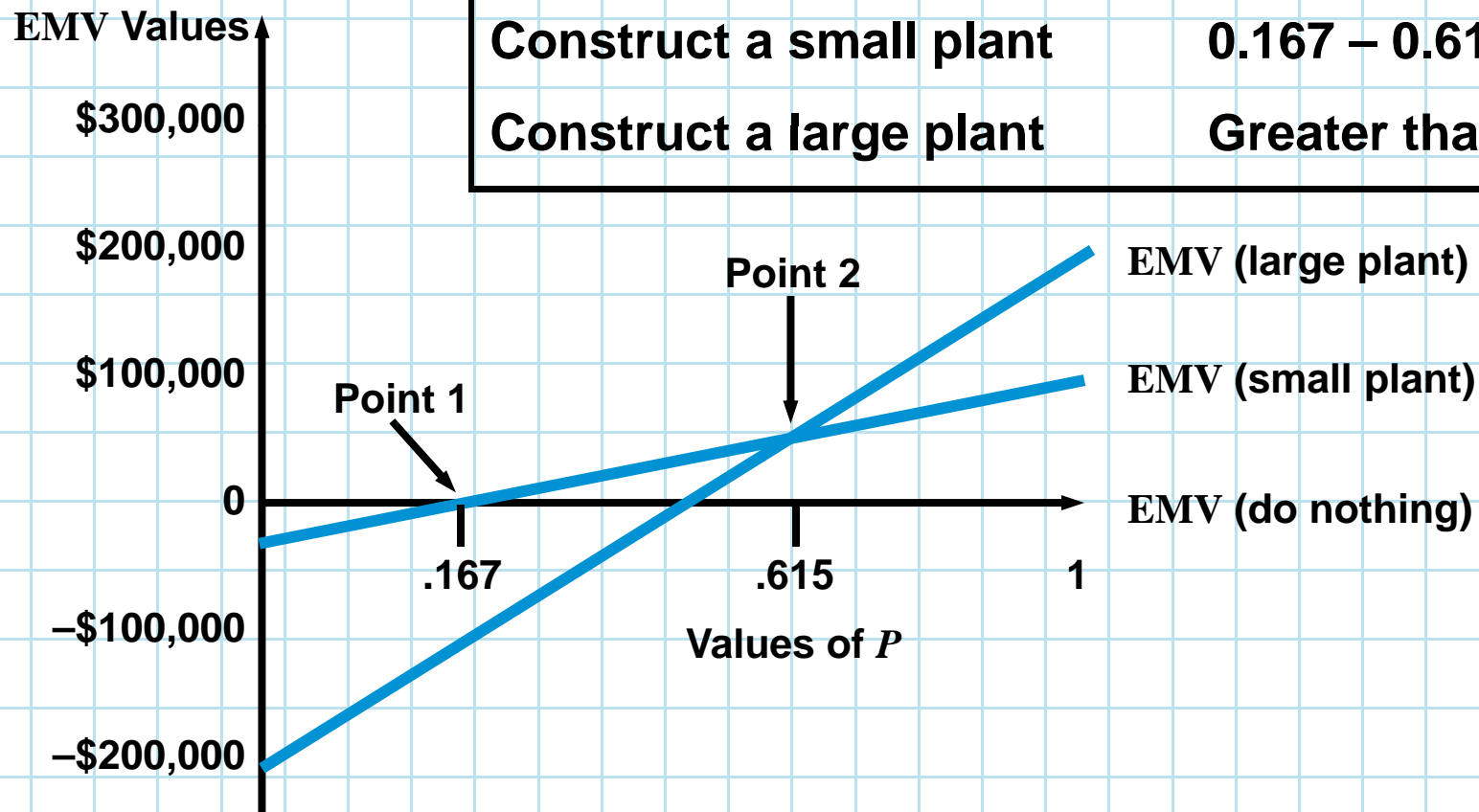


Figure 3.1

# Using Excel QM to Solve Decision Theory Problems

	A	B	C	D	E	F	
1	<b>Thompson Lumber</b>						
2							
3	<b>Decision Tables</b>						
4	Enter the profits or costs in the main body of the data table. Enter probabilities in the first row if you want						
5	to compute the expected value.						
6	Data			Results			
7	Profit	Favorable Market	Unfavorable Market	EMV	Minimum	Maximum	
8	Probability	0.5	0.5			Hurwicz	
9	Large Plant	200000	-180000	=SUMPRODUCT(B\$8:C\$8,B9:C9)	=MIN(B9:C9)	=MAX(B9:C9)	=G\$8*G9+(1-G\$8)*F9
10	Small plant	100000	-20000	=SUMPRODUCT(B\$8:C\$8,B10:C10)	=MIN(B10:C10)	=MAX(B10:C10)	=G\$8*G10+(1-G\$8)*F10
11	Do nothing	0	0	=SUMPRODUCT(B\$8:C\$8,B11:C11)	=MIN(B11:C11)	=MAX(B11:C11)	
12				=MAX(E9:E11)	=MAX(F9:F11)	=MAX(G9:G11)	=MAX(I9:I11)
13							
14	<b>Expected Value of Perfect Information</b>						
15	Column best	=MAX(B9:B11)	=MAX(C9:C11)	=SUMPRODUCT(B\$8:C\$8,B15:C15)	<- Expected value under certainty		
16				=E12	<- Best expected value		
17				=E15-E12	<- Expected value of perfect information		
18							
19	<b>Regret</b>						
20		=B7	=C7	Expected	Maximum		
21	=A8	=B8	=C8				
22	=A9	=B15 - B9	=C15 - C9	=SUMPRODUCT(B\$8:C\$8,B22:C22)	=MAX(B22:C22)		
23	=A10	=B15 - B10	=C15 - C10	=SUMPRODUCT(B\$8:C\$8,B23:C23)	=MAX(B23:C23)		
24	=A11	=B15 - B11	=C15 - C11	=SUMPRODUCT(B\$8:C\$8,B24:C24)	=MAX(B24:C24)		
25				=MIN(E22:E24)	=MIN(F22:F24)		
26							
27							
28							
29							

Compute the EMV for each alternative using the SUMPRODUCT function, the worst case using the MIN function, and the best case using the MAX function.

To calculate the EVPI, find the best outcome for each scenario.

Find the best outcome for each measure using the MAX function.

Use SUMPRODUCT to compute the product of the best outcomes by the probabilities and find the difference between this and the best expected value yielding the EVPI.

Program 3.1A

# Using Excel QM to Solve Decision Theory Problems

	A	B	C	D	E	F	G	H	I	J	
1	<b>Thompson Lumber</b>										
2											
3	<b>Decision Tables</b>										
4	Enter the profits or costs in the main body of the data table. Enter probabilities in the										
5	first row if you want to compute the expected value.										
6	<b>Data</b>			<b>Results</b>							
7	Profit	Favorable Market	Unfavorable Market		EMV	Minimum	Maximum		Hurwicz		
8	Probability	0.5	0.5					coefficient	0.8		
9	Large Plant	200000	-180000		10000	-180000	200000		124000		
10	Small plant	100000	-20000		40000	-20000	100000		76000		
11	Do nothing	0	0		0	0	0				
12				<b>Maximum</b>	<b>40000</b>	<b>0</b>	<b>200000</b>		<b>124000</b>		
13											
14	<b>Expected Value of Perfect Information</b>										
15	Column best	200000	0		100000	<-Expected value under certainty					
16					40000	<-Best expected value					
17					60000	<-Expected value of perfect information					
18											
19	<b>Regret</b>										
20		Favorable Market	Unfavorable Market		Expected	Maximum					
21	Probability	0.5	0.5								
22	Large Plant	0	180000		90000	180000					
23	Small plant	100000	20000		60000	100000					
24	Do nothing	200000	0		100000	200000					
25				<b>Minimum</b>	<b>60000</b>	<b>100000</b>					

Program 3.1B

# *Decision Trees*

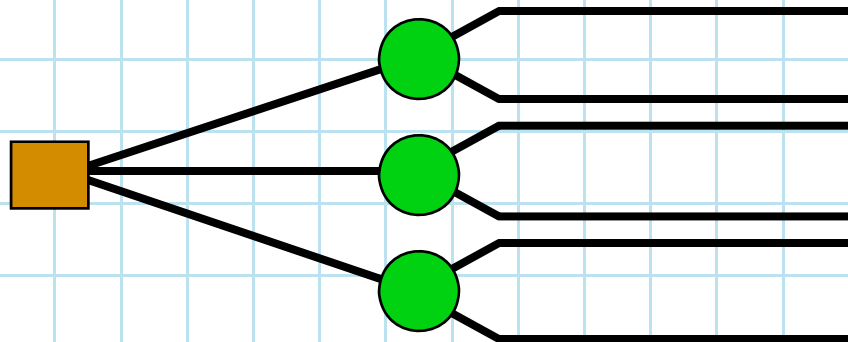
- Any problem that can be presented in a decision table can also be graphically represented in a *decision tree*
- Decision trees are most beneficial when a sequence of decisions must be made
- All decision trees contain *decision points* or *nodes* and *state-of-nature points* or *nodes*
  - A decision node from which one of several alternatives may be chosen
  - A state-of-nature node out of which one state of nature will occur

## *Five Steps to Decision Tree Analysis*

- 1. Define the problem**
- 2. Structure or draw the decision tree**
- 3. Assign probabilities to the states of nature**
- 4. Estimate payoffs for each possible combination of alternatives and states of nature**
- 5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node**

# *Structure of Decision Trees*

- **Trees start from left to right**
- **Represent decisions and outcomes in sequential order**
- **Squares represent decision nodes**
- **Circles represent states of nature nodes**
- **Lines or branches connect the decisions nodes and the states of nature**



# Thompson's Decision Tree

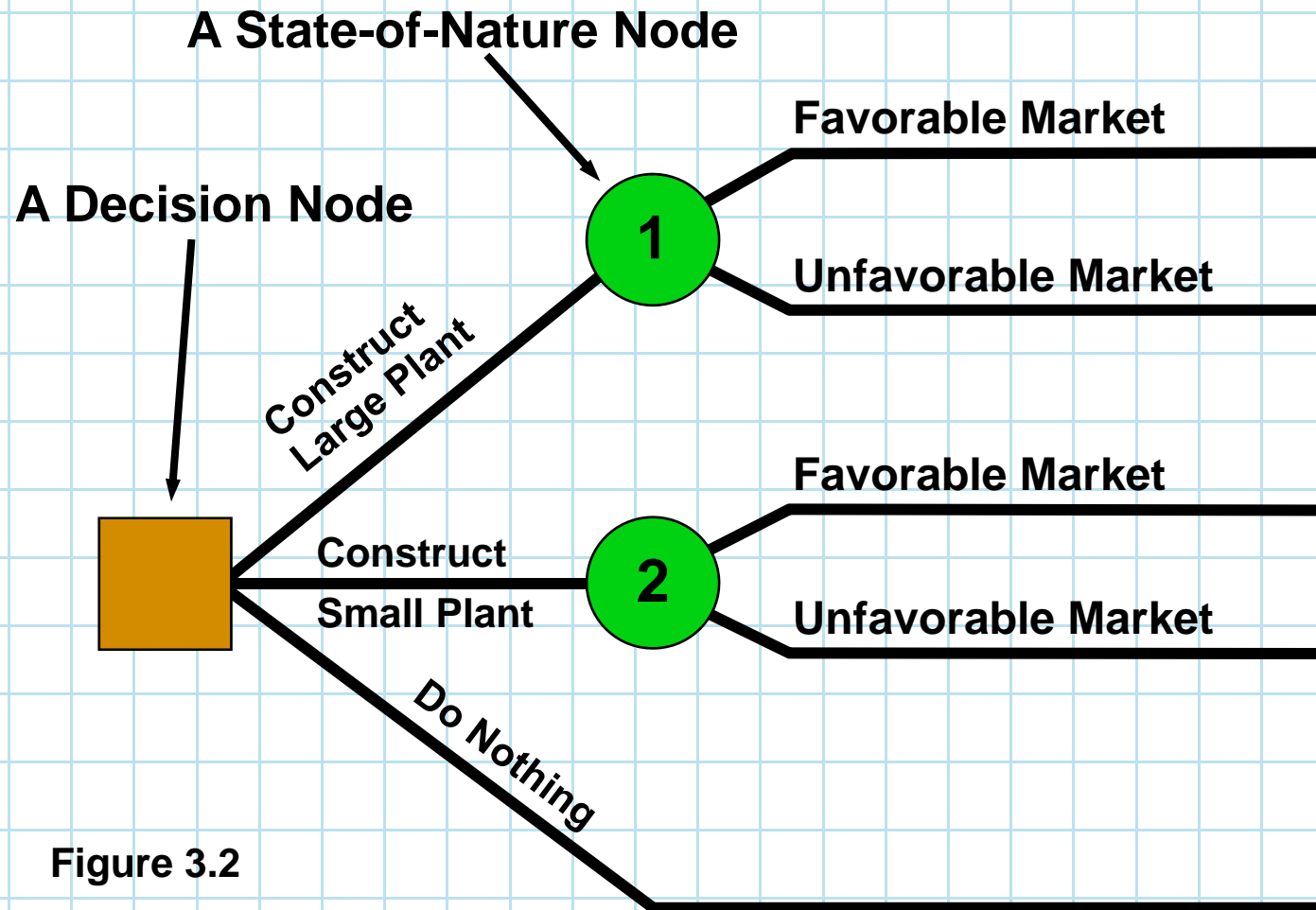


Figure 3.2

# Thompson's Decision Tree

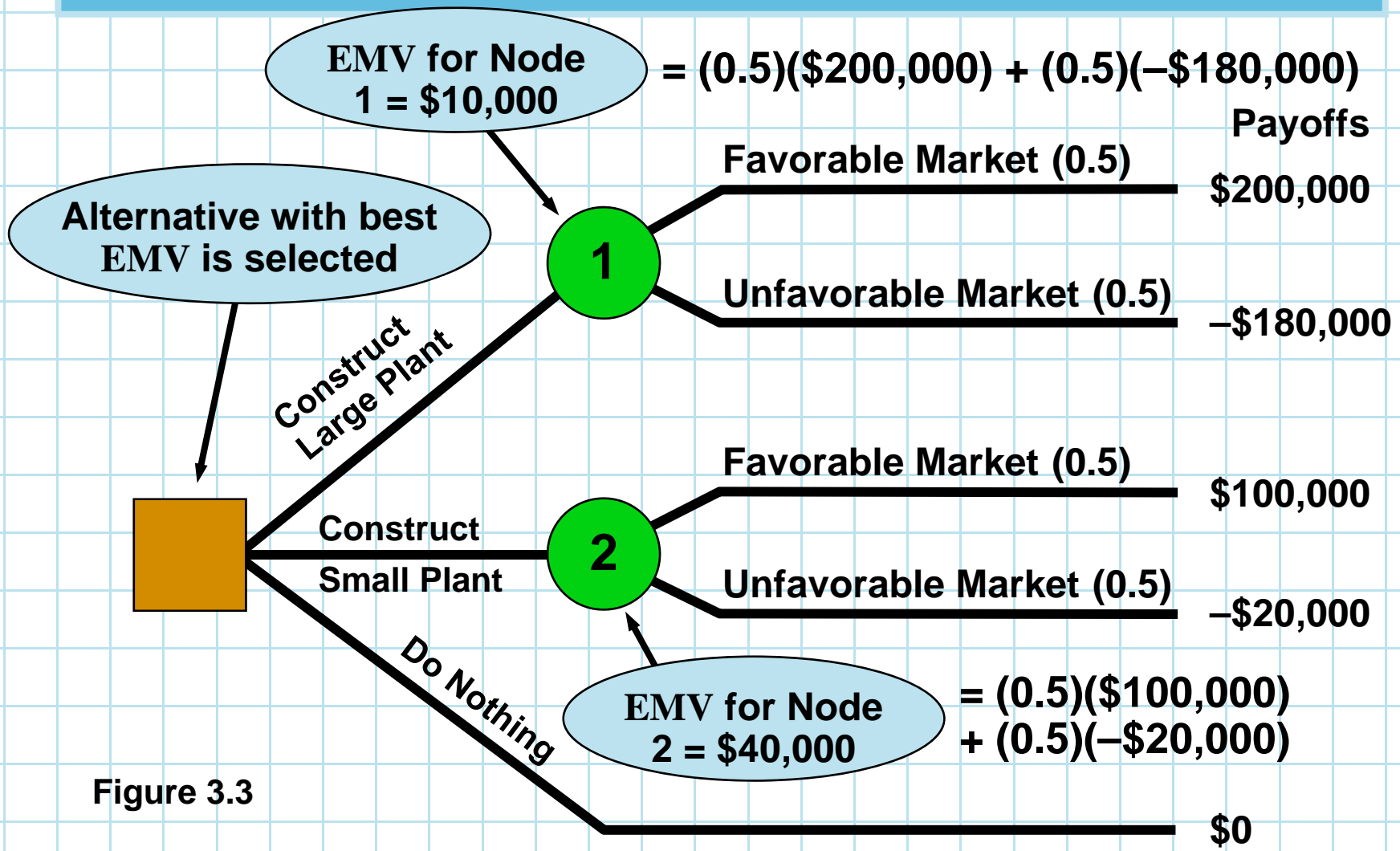


Figure 3.3

# Thompson's Complex Decision Tree

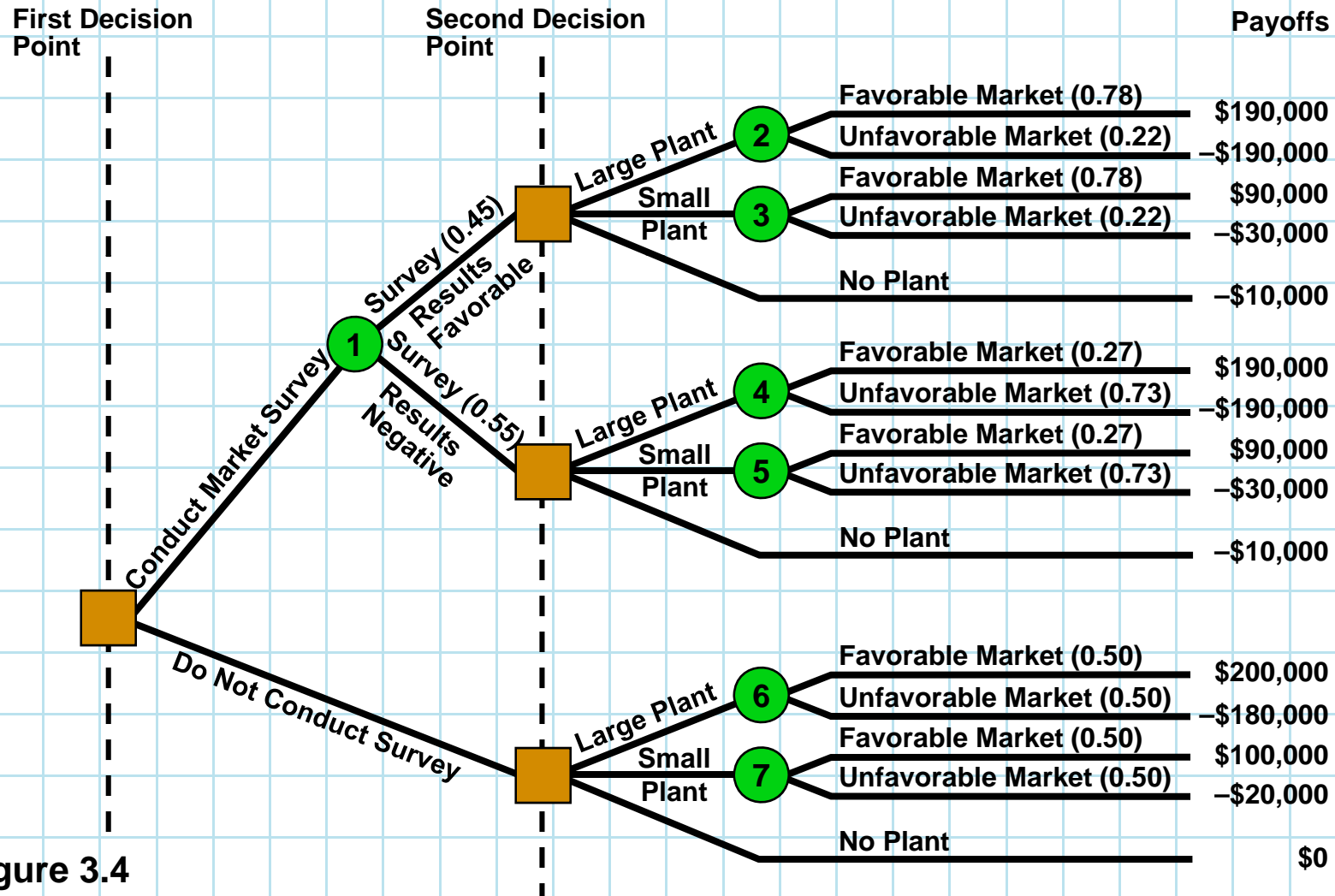


Figure 3.4

# *Thompson's Complex Decision Tree*

## **1. Given favorable survey results,**

$$\begin{aligned}\text{EMV}(\text{node 2}) &= \text{EMV}(\text{large plant} \mid \text{positive survey}) \\ &= (0.78)(\$190,000) + (0.22)(-\$190,000) = \$106,400\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 3}) &= \text{EMV}(\text{small plant} \mid \text{positive survey}) \\ &= (0.78)(\$90,000) + (0.22)(-\$30,000) = \$63,600\end{aligned}$$

$$\text{EMV for no plant} = -\$10,000$$

## **2. Given negative survey results,**

$$\begin{aligned}\text{EMV}(\text{node 4}) &= \text{EMV}(\text{large plant} \mid \text{negative survey}) \\ &= (0.27)(\$190,000) + (0.73)(-\$190,000) = -\$87,400\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 5}) &= \text{EMV}(\text{small plant} \mid \text{negative survey}) \\ &= (0.27)(\$90,000) + (0.73)(-\$30,000) = \$2,400\end{aligned}$$

$$\text{EMV for no plant} = -\$10,000$$

## ***Thompson's Complex Decision Tree***

### **3. Compute the expected value of the market survey,**

$$\begin{aligned}\text{EMV}(\text{node 1}) &= \text{EMV}(\text{conduct survey}) \\ &= (0.45)(\$106,400) + (0.55)(\$2,400) \\ &= \$47,880 + \$1,320 = \$49,200\end{aligned}$$

### **4. If the market survey is not conducted,**

$$\begin{aligned}\text{EMV}(\text{node 6}) &= \text{EMV}(\text{large plant}) \\ &= (0.50)(\$200,000) + (0.50)(-\$180,000) = \$10,000\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 7}) &= \text{EMV}(\text{small plant}) \\ &= (0.50)(\$100,000) + (0.50)(-\$20,000) = \$40,000\end{aligned}$$

$$\text{EMV for no plant} = \$0$$

### **5. Best choice is to seek marketing information**

# Thompson's Complex Decision Tree

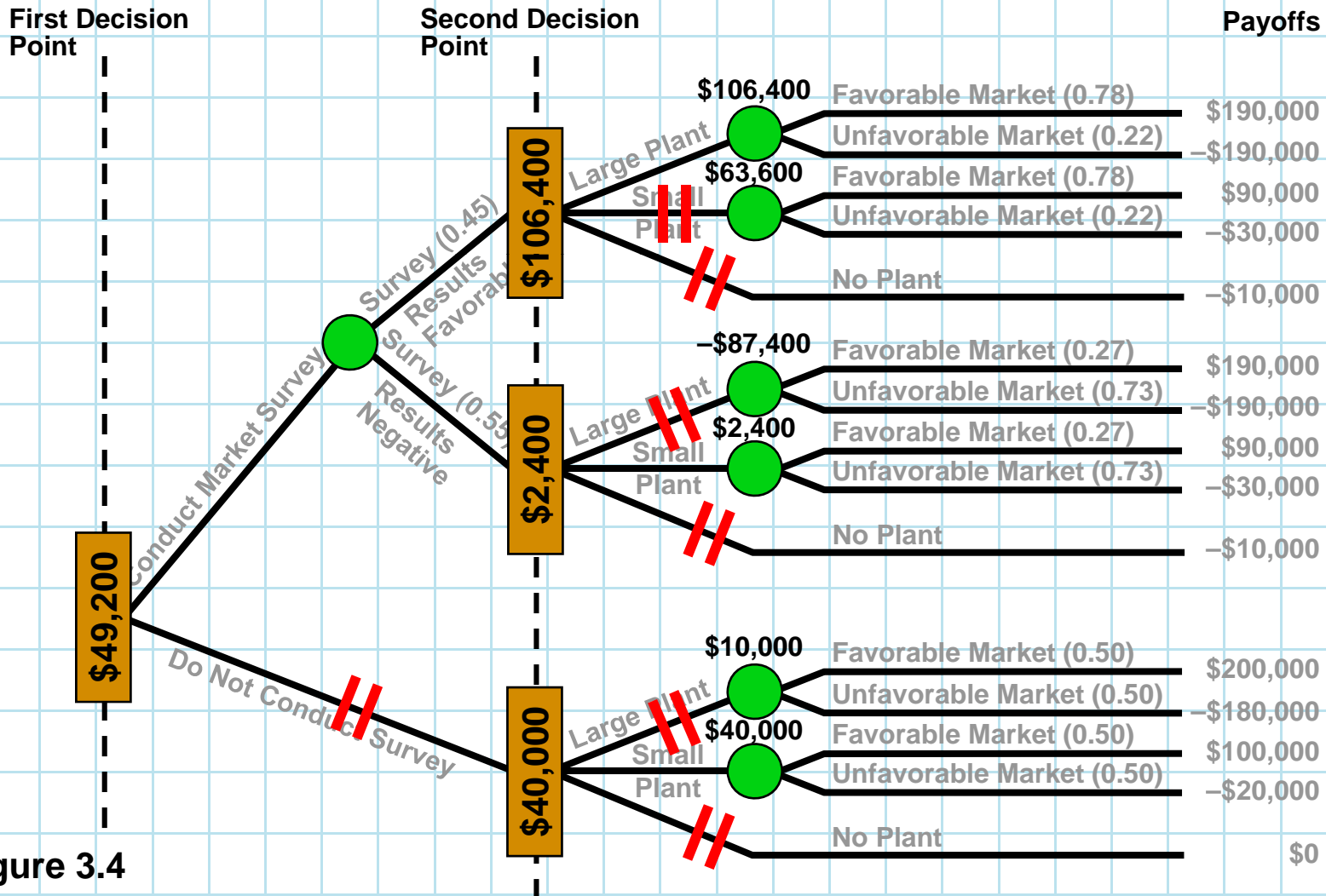


Figure 3.4

# *Expected Value of Sample Information*

- Thompson wants to know the actual value of doing the survey

$$\begin{aligned} \text{EVSI} &= \left( \begin{array}{l} \text{Expected value} \\ \text{with sample} \\ \text{information, assuming} \\ \text{no cost to gather it} \end{array} \right) - \left( \begin{array}{l} \text{Expected value} \\ \text{of best decision} \\ \text{without sample} \\ \text{information} \end{array} \right) \\ &= (\text{EV with sample information} + \text{cost}) \\ &\quad - (\text{EV without sample information}) \end{aligned}$$

$$\text{EVSI} = (\$49,200 + \$10,000) - \$40,000 = \$19,200$$

# ***Sensitivity Analysis***

- **How sensitive are the decisions to changes in the probabilities?**
  - **How sensitive is our decision to the probability of a favorable survey result?**
  - **That is, if the probability of a favorable result ( $p = .45$ ) were to change, would we make the same decision?**
  - **How much could it change before we would make a different decision?**

# ***Sensitivity Analysis***

$p$  = probability of a favorable survey result  
 $(1 - p)$  = probability of a negative survey result

$$\begin{aligned}\text{EMV}(\text{node 1}) &= (\$106,400)p + (\$2,400)(1 - p) \\ &= \$104,000p + \$2,400\end{aligned}$$

We are indifferent when the EMV of node 1 is the same as the EMV of not conducting the survey, \$40,000

$$\$104,000p + \$2,400 = \$40,000$$

$$\$104,000p = \$37,600$$

$$p = \$37,600 / \$104,000 = 0.36$$

# ***Bayesian Analysis***

- **Many ways of getting probability data**
- **It can be based on**
  - **Management's experience and intuition**
  - **Historical data**
  - **Computed from other data using Bayes' theorem**
- **Bayes' theorem incorporates initial estimates and information about the accuracy of the sources**
- **Allows the revision of initial estimates based on new information**

# ***Calculating Revised Probabilities***

- **In the Thompson Lumber case we used these four conditional probabilities**

**$P$  (favorable market(FM) | survey results positive) = 0.78**

**$P$  (unfavorable market(UM) | survey results positive) = 0.22**

**$P$  (favorable market(FM) | survey results negative) = 0.27**

**$P$  (unfavorable market(UM) | survey results negative) = 0.73**

- **The prior probabilities of these markets are**

**$P$  (FM) = 0.50**

**$P$  (UM) = 0.50**

# Calculating Revised Probabilities

- Through discussions with experts Thompson has learned the following
- He can use this information and Bayes' theorem to calculate posterior probabilities

RESULT OF SURVEY	STATE OF NATURE	
	FAVORABLE MARKET (FM)	UNFAVORABLE MARKET (UM)
Positive (predicts favorable market for product)	$P(\text{survey positive} \mid \text{FM}) = 0.70$	$P(\text{survey positive} \mid \text{UM}) = 0.20$
Negative (predicts unfavorable market for product)	$P(\text{survey negative} \mid \text{FM}) = 0.30$	$P(\text{survey negative} \mid \text{UM}) = 0.80$

Table 3.11

# Calculating Revised Probabilities

- Recall Bayes' theorem is

$$P(A | B) = \frac{P(B | A) \times P(A)}{P(B | A) \times P(A) + P(B | A') \times P(A')}$$

where

$A, B$  = any two events

$A'$  = complement of  $A$

For this example,  $A$  will represent a favorable market and  $B$  will represent a positive survey

# Calculating Revised Probabilities

## ■ $P(FM \mid \text{survey positive})$

$$\begin{aligned} &= \frac{P(\text{survey positive} \mid FM) \times P(FM)}{P(\text{survey positive} \mid FM) \times P(FM) + P(\text{survey positive} \mid UM) \times P(UM)} \\ &= \frac{(0.70)(0.50)}{(0.70)(0.50) + (0.20)(0.50)} = \frac{0.35}{0.45} = 0.78 \end{aligned}$$

## ■ $P(UM \mid \text{survey positive})$

$$\begin{aligned} &= \frac{P(\text{survey positive} \mid UM) \times P(UM)}{P(\text{survey positive} \mid UM) \times P(UM) + P(\text{survey positive} \mid FM) \times P(FM)} \\ &= \frac{(0.20)(0.50)}{(0.20)(0.50) + (0.70)(0.50)} = \frac{0.10}{0.45} = 0.22 \end{aligned}$$

# Calculating Revised Probabilities

STATE OF NATURE	CONDITIONAL PROBABILITY $P(\text{SURVEY POSITIVE} \mid \text{STATE OF NATURE})$	PRIOR PROBABILITY	POSTERIOR PROBABILITY	
			JOINT PROBABILITY	$P(\text{STATE OF NATURE} \mid \text{SURVEY POSITIVE})$
FM	0.70	X 0.50	= 0.35	$0.35/0.45 = 0.78$
UM	0.20	X 0.50	= 0.10	$0.10/0.45 = 0.22$
$P(\text{survey results positive}) =$			<u>0.45</u>	<u>1.00</u>

Table 3.12

# Calculating Revised Probabilities

## ■ $P(FM \mid \text{survey negative})$

$$\begin{aligned} &= \frac{P(\text{survey negative} \mid FM) \times P(FM)}{P(\text{survey negative} \mid FM) \times P(FM) + P(\text{survey negative} \mid UM) \times P(UM)} \\ &= \frac{(0.30)(0.50)}{(0.30)(0.50) + (0.80)(0.50)} = \frac{0.15}{0.55} = 0.27 \end{aligned}$$

## ■ $P(UM \mid \text{survey negative})$

$$\begin{aligned} &= \frac{P(\text{survey negative} \mid UM) \times P(UM)}{P(\text{survey negative} \mid UM) \times P(UM) + P(\text{survey negative} \mid FM) \times P(FM)} \\ &= \frac{(0.80)(0.50)}{(0.80)(0.50) + (0.30)(0.50)} = \frac{0.40}{0.55} = 0.73 \end{aligned}$$

# Calculating Revised Probabilities

STATE OF NATURE	CONDITIONAL PROBABILITY $P(\text{SURVEY NEGATIVE} \mid \text{STATE OF NATURE})$	PRIOR PROBABILITY	POSTERIOR PROBABILITY	
			JOINT PROBABILITY	$P(\text{STATE OF NATURE} \mid \text{SURVEY NEGATIVE})$
FM	0.30	X 0.50	= 0.15	$0.15/0.55 = 0.27$
UM	0.80	X 0.50	= 0.40	$0.40/0.55 = 0.73$
$P(\text{survey results positive}) =$			<u>0.55</u>	<u>1.00</u>

Table 3.13

# *Potential Problems Using Survey Results*

- **We can not always get the necessary data for analysis**
- **Survey results may be based on cases where an action was taken**
- **Conditional probability information may not be as accurate as we would like**

# *Utility Theory*

- Monetary value is not always a true indicator of the overall value of the result of a decision
- The overall value of a decision is called *utility*
- Rational people make decisions to maximize their utility

# Utility Theory

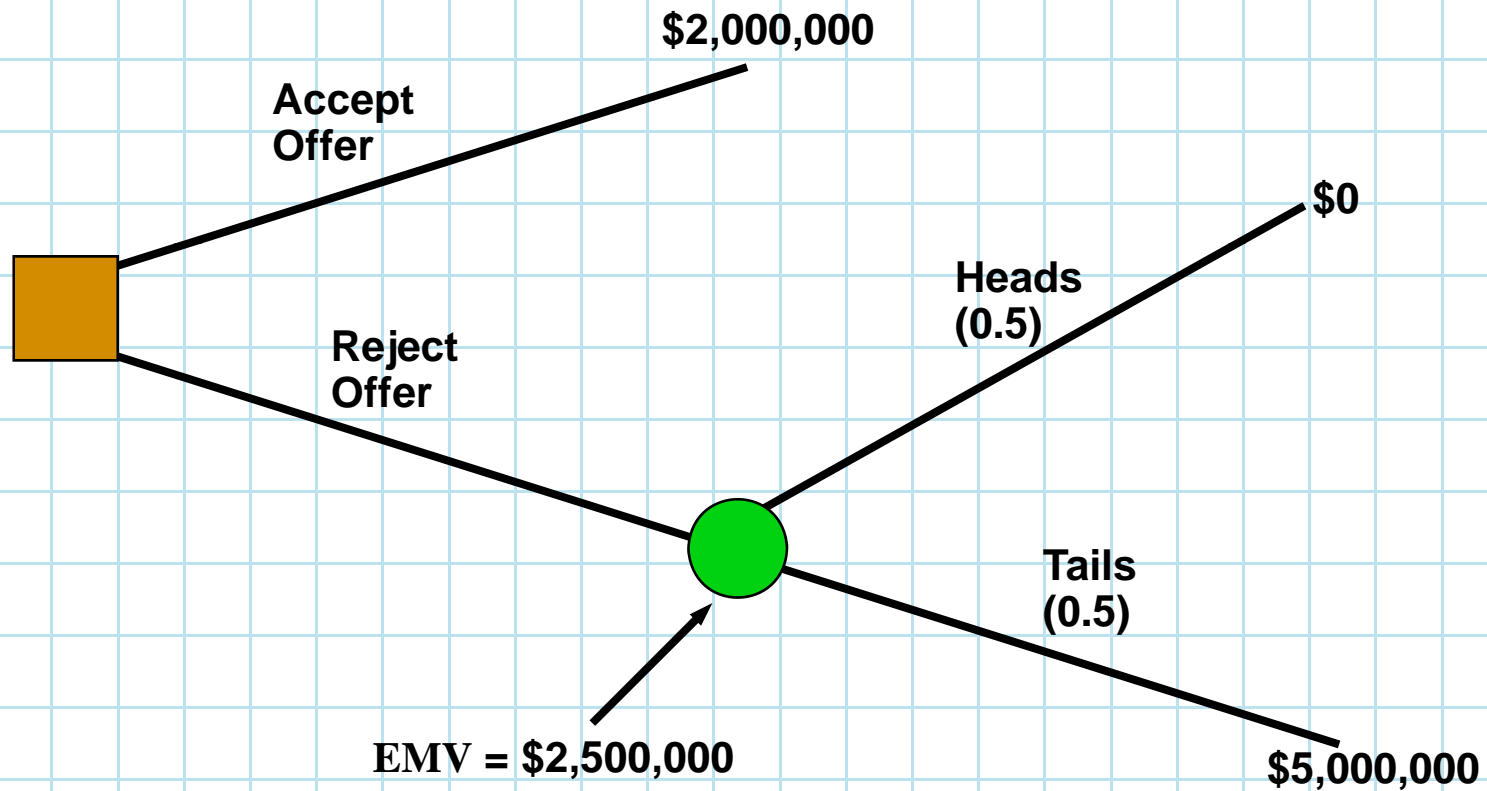


Figure 3.6

# Utility Theory

- **Utility assessment** assigns the worst outcome a utility of 0, and the best outcome, a utility of 1
- A **standard gamble** is used to determine utility values
- When you are indifferent, the utility values are equal

Expected utility of alternative 2 = Expected utility of alternative 1

Utility of other outcome =  $(p)(\text{utility of best outcome, which is 1})$   
+  $(1 - p)(\text{utility of the worst outcome, which is 0})$

Utility of other outcome =  $(p)(1) + (1 - p)(0) = p$

# Standard Gamble

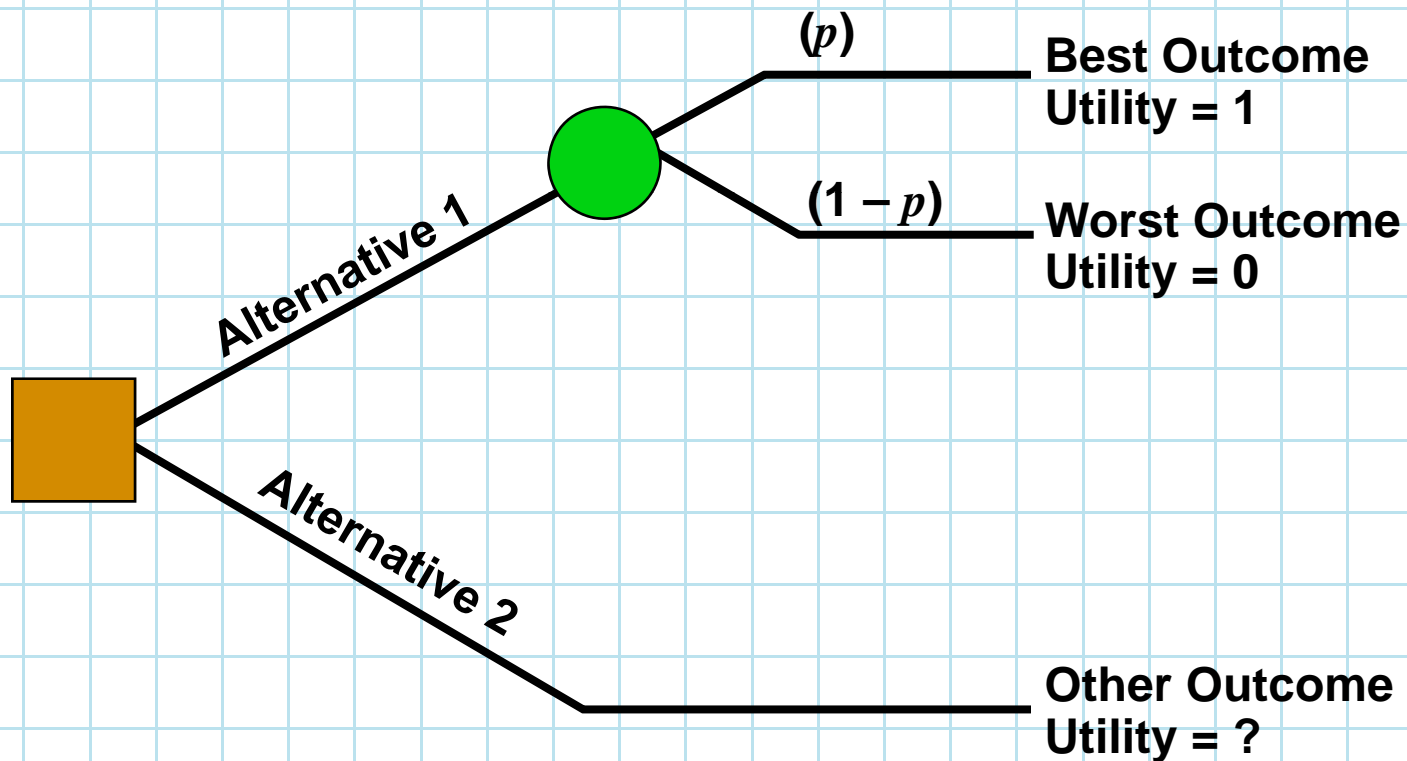


Figure 3.7

# *Investment Example*

- Jane Dickson wants to construct a utility curve revealing her preference for money between \$0 and \$10,000
- A utility curve plots the utility value versus the monetary value
- An investment in a bank will result in \$5,000
- An investment in real estate will result in \$0 or \$10,000
- Unless there is an 80% chance of getting \$10,000 from the real estate deal, Jane would prefer to have her money in the bank
- So if  $p = 0.80$ , Jane is indifferent between the bank or the real estate investment

# Investment Example

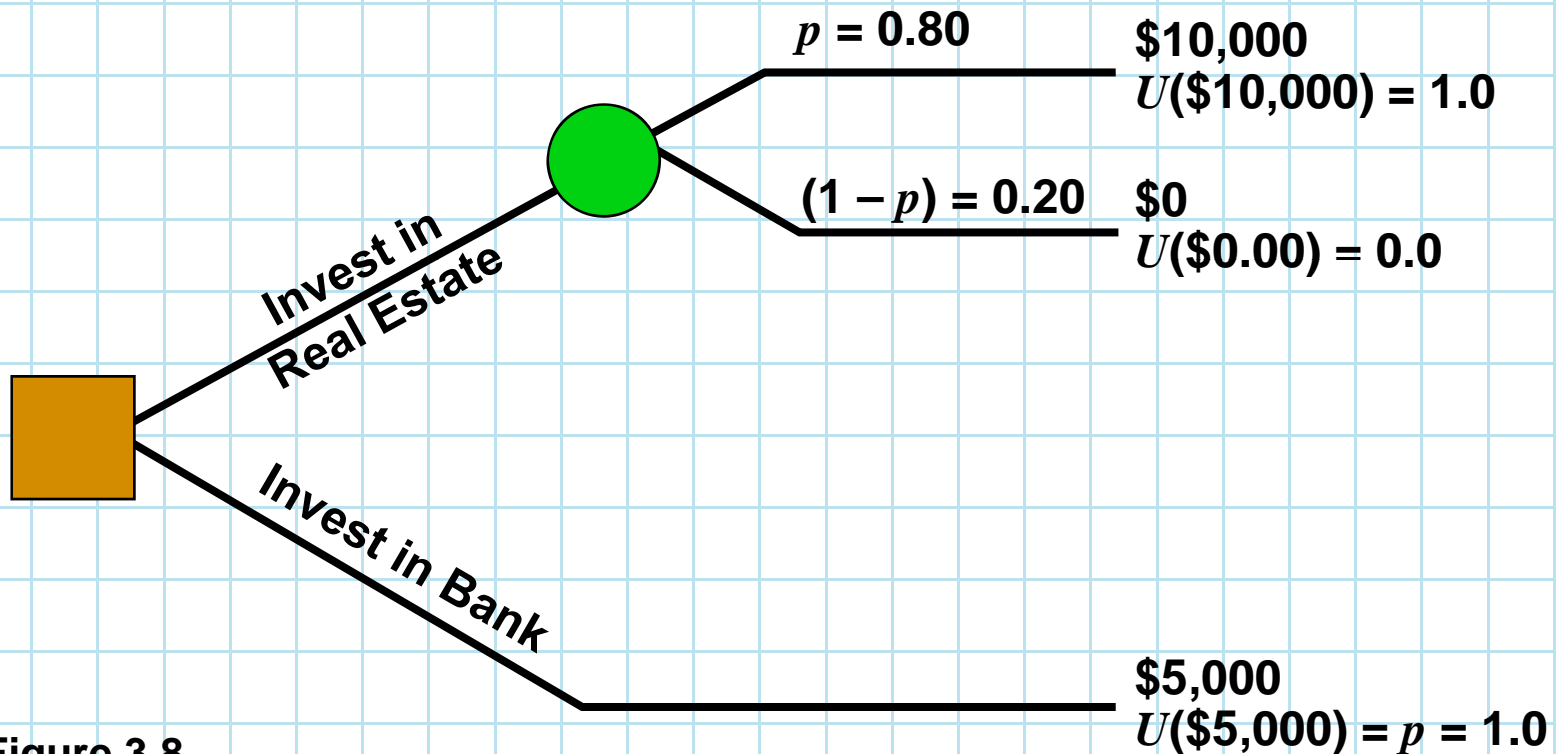


Figure 3.8

$$\begin{aligned} \text{Utility for } \$5,000 = U(\$5,000) &= pU(\$10,000) + (1 - p)U(\$0) \\ &= (0.8)(1) + (0.2)(0) = 0.8 \end{aligned}$$

# *Investment Example*

- We can assess other utility values in the same way
- For Jane these are

**Utility for \$7,000 = 0.90**

**Utility for \$3,000 = 0.50**

- Using the three utilities for different dollar amounts, she can construct a utility curve

# Utility Curve

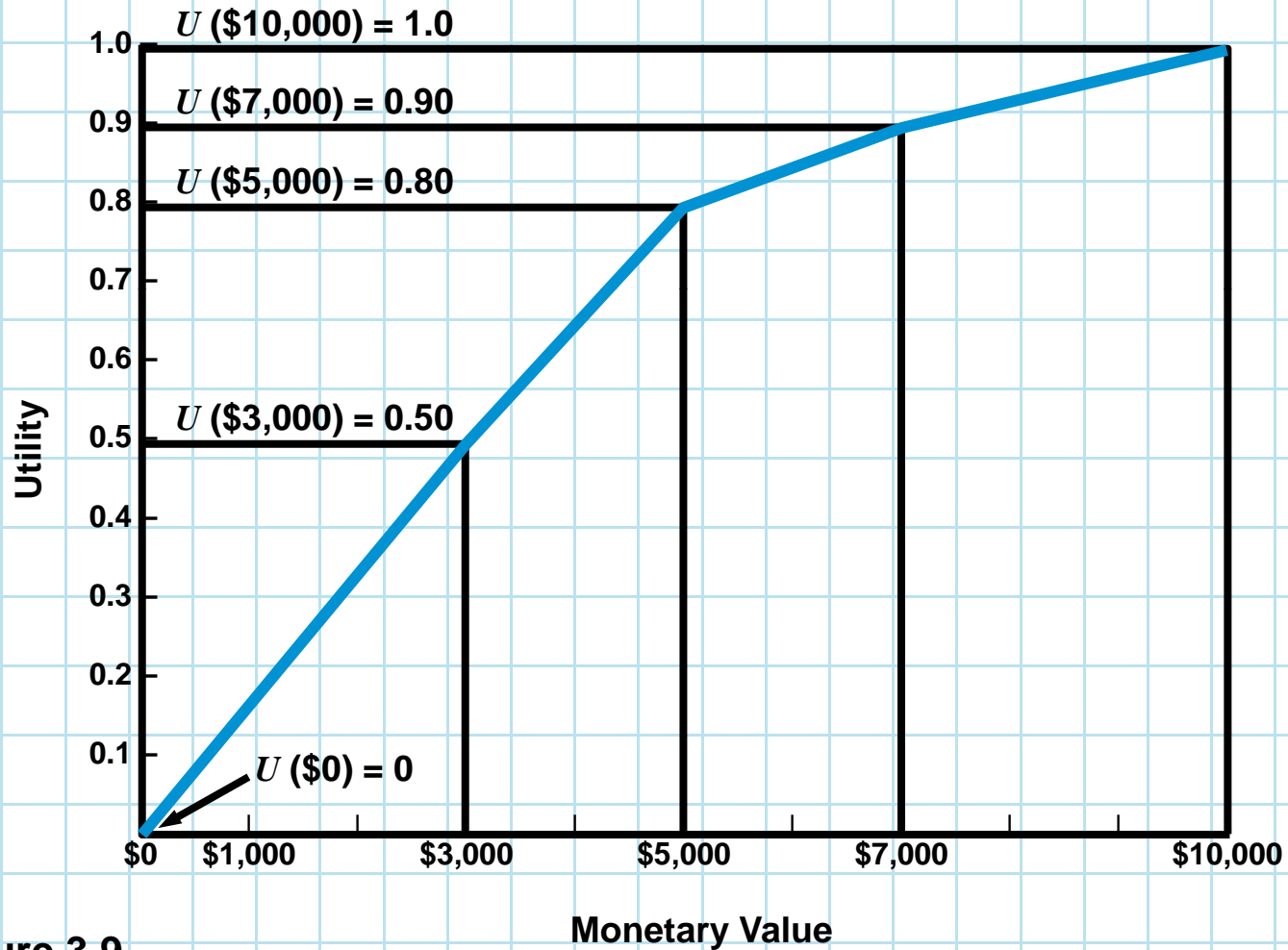


Figure 3.9

# *Utility Curve*

- **Jane's utility curve is typical of a risk avoider**
  - **A risk avoider gets less utility from greater risk**
  - **Avoids situations where high losses might occur**
  - **As monetary value increases, the utility curve increases at a slower rate**
  - **A risk seeker gets more utility from greater risk**
  - **As monetary value increases, the utility curve increases at a faster rate**
  - **Someone who is indifferent will have a linear utility curve**

# Utility Curve

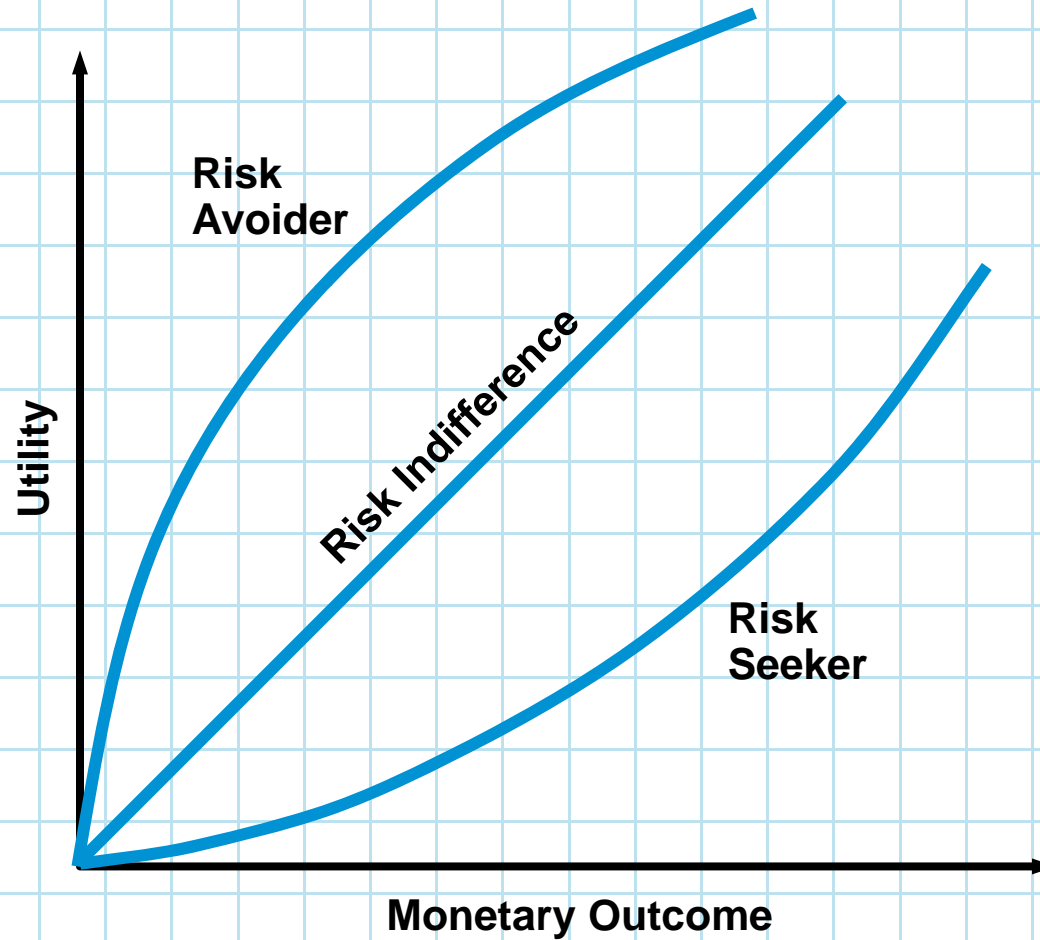


Figure 3.10

## ***Utility as a Decision-Making Criteria***

- **Once a utility curve has been developed it can be used in making decisions**
- **Replace monetary outcomes with utility values**
- **The expected utility is computed instead of the EMV**

## *Utility as a Decision-Making Criteria*

- **Mark Simkin loves to gamble**
- **He plays a game tossing thumbtacks in the air**
- **If the thumbtack lands point up, Mark wins \$10,000**
- **If the thumbtack lands point down, Mark loses \$10,000**
- **Should Mark play the game (alternative 1)?**

# Utility as a Decision-Making Criteria

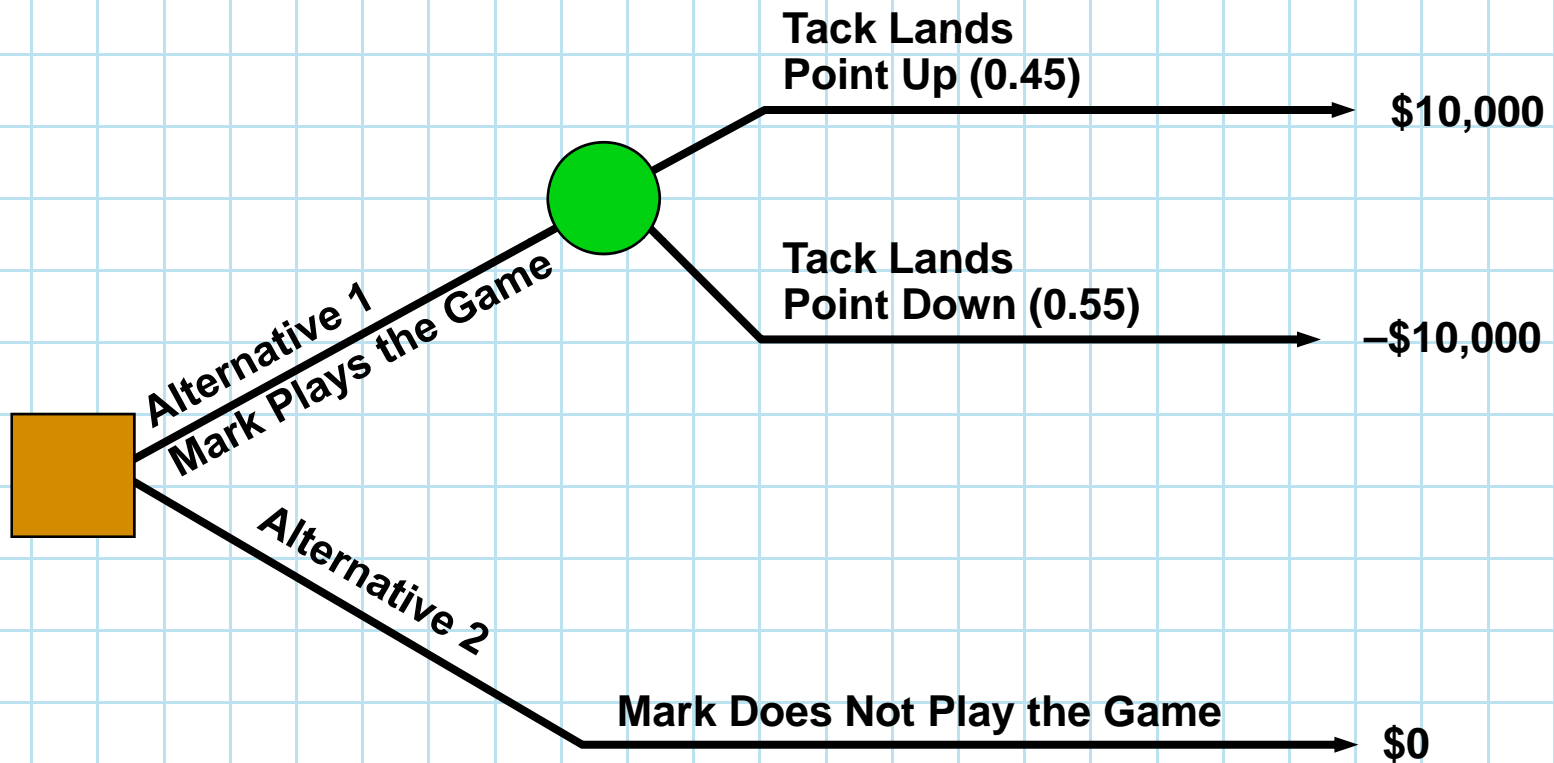


Figure 3.11

# ***Utility as a Decision-Making Criteria***

## ■ **Step 1– Define Mark’s utilities**

$$U (-\$10,000) = 0.05$$

$$U (\$0) = 0.15$$

$$U (\$10,000) = 0.30$$

## ■ **Step 2 – Replace monetary values with utility values**

$$\begin{aligned} E(\text{alternative 1: play the game}) &= (0.45)(0.30) + (0.55)(0.05) \\ &= 0.135 + 0.027 = 0.162 \end{aligned}$$

$$E(\text{alternative 2: don't play the game}) = 0.15$$

# Utility as a Decision-Making Criteria

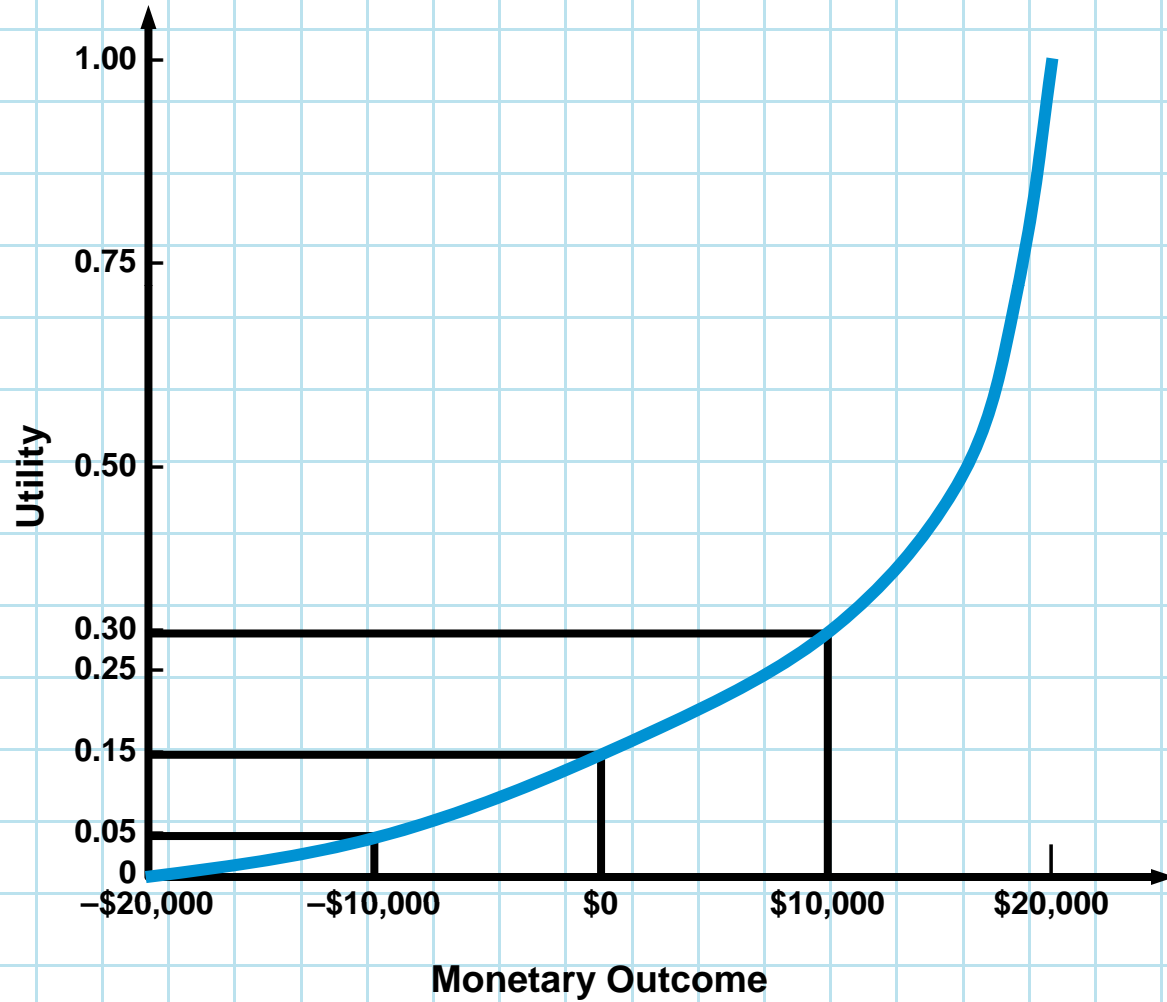


Figure 3.12

# Utility as a Decision-Making Criteria

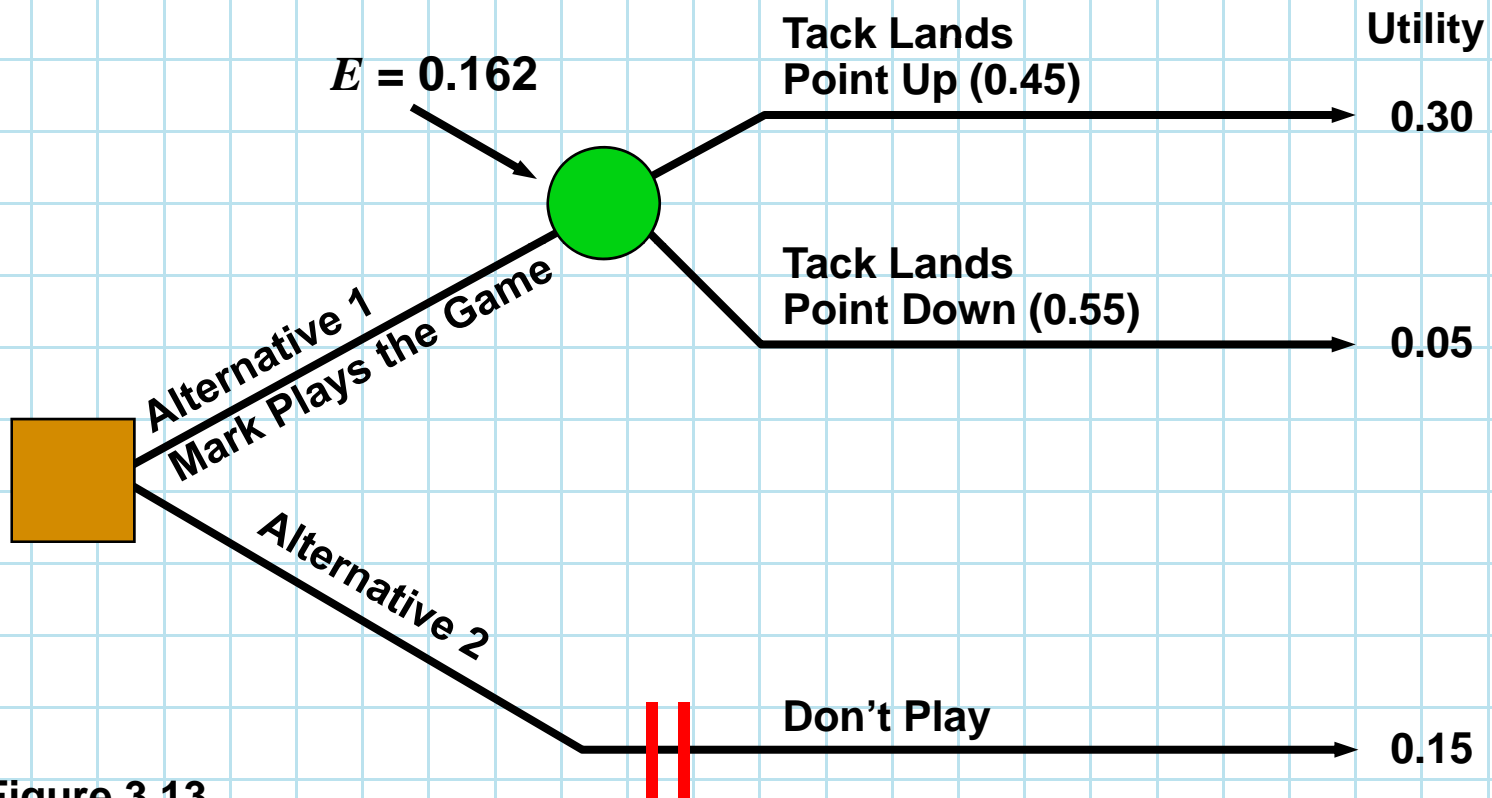


Figure 3.13