

89.304 - IGNEOUS & METAMORPHIC PETROLOGY
DENSITY & VISCOSITY OF MAGMAS

I. Density

The *density* (mass/volume) of a magma is an important parameter which plays a role in a number of aspects of magma behavior and evolution. For example, during fractional crystallization the rate at which crystals settle (or rise) through a magma is a function of the density contrast between the magma and the crystals, the size of the crystals, and the viscosity of the magma. Additionally, the contrast in magma density is important in governing the rate of convective overturn in a magma chamber and magma mixing. Additionally, the contrast in density between a magma column and the surrounding country rock will determine the height to which a magma column will rise.

The physical measurement of magma density is rather difficult. Hence, a variety of methods have been developed to estimate or determine magma density without having to directly determine the density of the liquid magma.

Density estimated from average volume increase during melting. From observation we know that the volume expansion on melting for most rocks is about 10%. Hence, most magmas have a density about 90% that of the equivalent solid rock. This is a relatively crude, but sometimes adequate, estimate.

Density determined from quenched glass at room temperature. Because glass is a supercooled liquid, the high-temperature density can be calculated from the room-temperature density if the coefficient of thermal expansion is known. The density of the liquid at temperature T can be calculated as follows

$$\rho_T = \rho_1[1 - \alpha(T - T_1)]$$

where ρ_T = the density at temperature T, ρ_1 = the density at room temperature (or the temperature at which the glass density was measured), α = the coefficient of thermal expansion, T = the temperature at which the density is calculated, and T_1 = the temperature at which the density is measured.

1. A sample of granite with density 2670 kg m^{-3} is fused and quenched to a glass that has a density of 2450 kg m^{-3} at 25°C . The coefficient of expansion, α , for the glass is $2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$. Calculate the density of the magma at 1025°C , and compare this with the value obtained by simply taking 90% of the density of the rock.

Density calculated from molar volumes of constituent oxides. Bottinga and Weill (1970) developed a model which enables one to calculate the magma density from the composition of the magma. Density is written as

$$\rho = \frac{M}{V}$$

where M = mass and V = volume. In Bottinga and Weill's model

$$\rho = \frac{\sum_{i=1}^n (X_i M_i)}{\sum_{i=1}^n (X_i \bar{V}_i)}$$

where X_i = the mole fraction of component i , M_i = the gram formula weight of component i , and \bar{V}_i = the partial molar volume of component i . The important parameter is \bar{V}_i , measured in $\text{m}^3/(\text{mole} \times 10^6)$, which varies as a function of temperature. Note that what \bar{V}_i represents is the number of m^3 occupied by 1 mole of a particular oxide. Therefore, if we determine the fractional mass of each oxide component in the melt, and the volume that each oxide occupies, the summation for all oxides gives the density of the magma.

The table below gives the partial molar volumes and coefficients of expansion for common oxide components of silicate liquids at 1400°C. The partial molar volumes at temperatures other than 1400°C can be calculated from the relationship

$$V_T = V_1[1 + \alpha(T - T_1)]$$

where V_T = the volume at temperature T , V_1 = the volume at 1400°C, α = the coefficient of thermal expansion, T = the temperature at which the volume is calculated, and $T_1 = 1400^\circ\text{C}$.

Oxide	Molar Volume $V_i (\text{m}^3 \text{mol}^{-1}) \times 10^{-6}$	Coefficient of expansion $\alpha (^\circ\text{C}^{-1}) \times 10^{-5}$
SiO ₂	26.75	0.1
TiO ₂	22.45	37.1
Al ₂ O ₃	37.80	2.6
Fe ₂ O ₃	44.40	32.1
FeO	13.94	34.7
MgO	12.32	12.2
CaO	16.59	16.7
Na ₂ O	29.03	25.9
K ₂ O	46.30	35.9

The partial molar volume of water can be calculated as follows:

$$\overline{V}_{\text{H}_2\text{O}} = 24.7 + 7.0 \times 10^{-3}(T - 900) - 1.5P + 8.4 \times 10^{-2}P^2 - 1.4 \times 10^{-3}P^2$$

where $\overline{V}_{\text{H}_2\text{O}} = \text{m}^3/(\text{mole} \times 10^6)$, $T = ^\circ\text{C}$, and $P = \text{kilobars}$. Note that the model of Bottinga and Weill ignores the effect of pressure on all the oxide components except water. This is a reasonable simplification for magmas which reside in the crust, but at mantle depths pressure should be considered.

2. Calculate the densities of the magmas Rhyolite 1 (800 °C), Rhyolite 2 (800 °C), and the Basalt at 1000 °C and 1100 °C. The relevant data and space for the calculations are found in the table on page 4.

3. Based on the calculated densities, what general conclusions can you make about the effect of composition, water content, and temperature on magma density?

Oxide	Rhyolite 1	Molecular proportion	Mole fraction	Rhyolite 2	Molecular proportion	Mole fraction	Basalt	Molecular proportion	Mole fraction	Molecular weights
SiO ₂	72.82			72.82			49.20			60.08
TiO ₂	0.28			0.28			1.84			79.90
Al ₂ O ₃	13.27			13.27			15.74			101.96
Fe ₂ O ₃	1.48			1.48			3.79			159.69
FeO	1.11			1.11			7.13			71.85
MnO	0.06			0.06			0.20			70.94
MgO	0.39			0.39			6.73			40.31
CaO	1.14			1.14			9.47			56.08
Na ₂ O	3.55			3.55			2.91			61.98
K ₂ O	4.30			4.30			1.10			94.20
H ₂ O				1.10						18.00

Rhyolite 1:
(T = 800°C) $\rho = \frac{\sum_{i=1}^n (X_i M_i)}{\sum_{i=1}^n (X_i \bar{V}_i)} =$

Rhyolite 2:
(T = 800°C)
(P = 1kbar) $\rho = \frac{\sum_{i=1}^n (X_i M_i)}{\sum_{i=1}^n (X_i \bar{V}_i)} =$

$$\bar{V}_{H_2O} = 24.7 + 7.0 \times 10^{-3}(T - 900) - 1.5P + 8.4 \times 10^{-2}P^2 - 1.4 \times 10^{-3}P^3 =$$

Basalt 1:
(T = 1000°C) $\rho = \frac{\sum_{i=1}^n (X_i M_i)}{\sum_{i=1}^n (X_i \bar{V}_i)} =$

Basalt 2:
(T = 1100°C) $\rho = \frac{\sum_{i=1}^n (X_i M_i)}{\sum_{i=1}^n (X_i \bar{V}_i)} =$

II. Viscosity

Viscosity is a measure of the internal resistance of a liquid body to flow. Viscosity plays a role in determining the style of magmatic emplacement and irruption, the rate at which crystals or other solids settle through a melt, and the rate of diffusion of ions through a melt. The *coefficient of viscosity* is the ratio of the applied shear stress to the time rate of shear strain. The viscosity is said to be *Newtonian* if this coefficient is independent of the magnitude of the shear stress and strain. The structure of a silicate melt (which is a function of its composition), temperature, and pressure determine the viscosity of a melt.

Unlike density, it has proven very difficult to develop a model for the calculation of magma viscosity. However, it is possible to determine the viscosity of melts (once the density is known) by measuring the rate at which spheres of known size and density settle through the melt. The governing principle is Stoke's Law

$$v = \frac{2g\Delta\rho r^2}{9\eta}$$

where v = settling velocity (m s^{-1}), g = acceleration due to gravity (9.8 m s^{-2}), $\Delta\rho$ = density contrast between the sphere and the melt (kg m^{-3}), r = the radius of the sphere (m), and η = dynamic viscosity (Pa s). Given the density of the sphere, the magma, and the radius of the sphere, we can measure the settling velocity of the sphere and from this velocity calculate the viscosity of the magma.

4. Using Stoke's Law, calculate the viscosities of the four magmas considered in question 1. The radius of the sphere is 0.01 m and its density is 5000 kg m^{-3} .

Magma	Density (kg m^{-3})	Settling velocity (m s^{-1})	Viscosity (Pa s)
Rhyolite 1		5.71×10^{-9}	
Rhyolite 2		5.77×10^{-8}	
Basalt (1000 °C)		5.07×10^{-5}	
Basalt (1100 °C)		5.05×10^{-4}	

5. Based on the calculated viscosities, what general conclusions can you make about the effect of composition, water content, and temperature on the viscosity of magmas?

III. Melt Polymerization

A silicate melt can be viewed as a material that has a short-range structural order. The structure consists of certain cations tetrahedrally coordinated with O^{2-} ions forming tetrahedra that are then linked directly to other tetrahedra or separated from other tetrahedra by network modifying cations. The degree to which the silica tetrahedra are linked is referred to as *polymerization*. For example, a melt formed from quartz would be highly polymerized with virtually all the tetrahedra interconnected. A measure of the degree of polymerization is the ratio of *tetrahedrally coordinated cations* (T) to *nonbridging oxygen ions* (NBO), i.e., oxygen ions that are directly bonded to tetrahedrally coordinated cations. The larger this ratio, the less polymerized is the melt.

The NBO/T ratio is calculated as follows:

1. Convert the chemical analysis of the rock to atomic proportions
 2. Assign Si, Ti, and P to T
 3. Assign Al to T unless $Al > Na + K + 2Ca + 2Mg$. In this case, $Al \text{ in T} = Na + K + 2Ca + 2Mg$
 4. $Fe^{3+} \text{ in T} = (Na + K) - Al \leq Fe^{3+}$
 5. Sum all T cations = T
 6. $NBO = 2O - 4T$
 7. Calculate NBO/T
6. Calculate the NBO/T ratio for Rhyolite 1 and Basalt. Note that you can obtain the atomic proportions by multiplying the molecular proportions by the number of ions in the oxide formula. For example, in the case of Al_2O_3 , $Al = 2 \times \text{molecular proportion } Al_2O_3$ and $O = 3 \times \text{molecular proportion } Al_2O_3$. What is the relationship between the degree of polymerization and the viscosity of the melts?