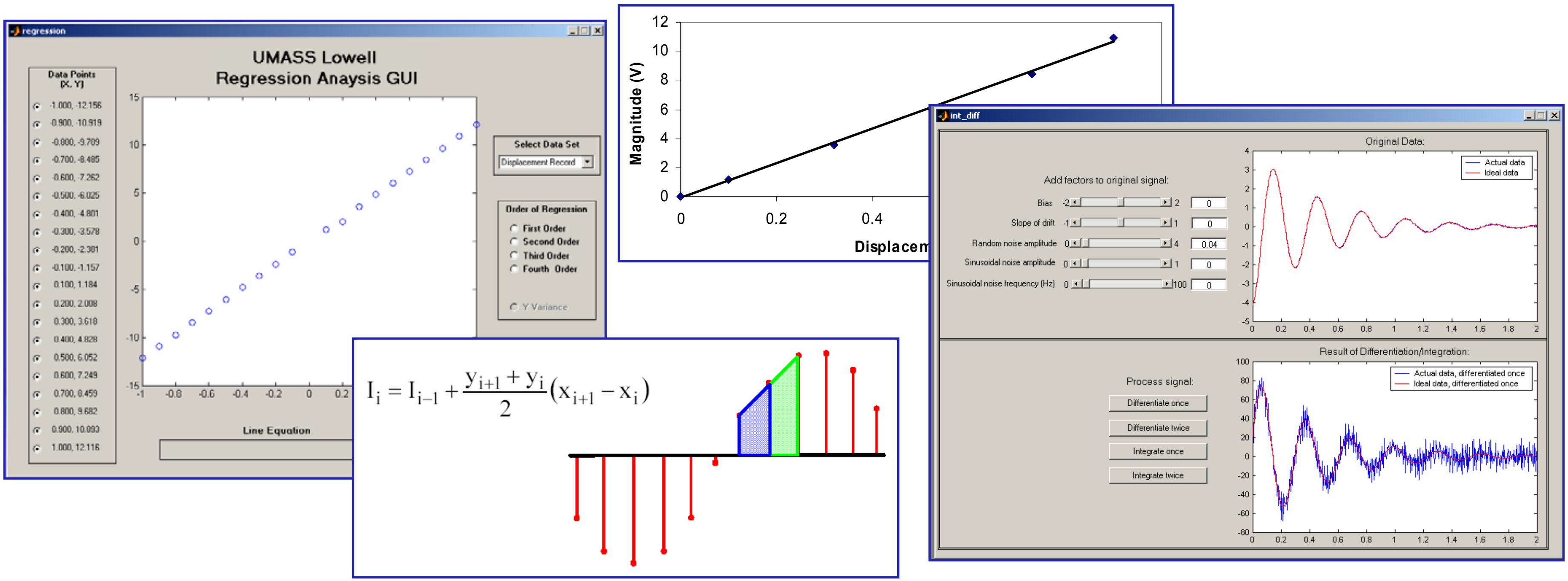
2005 American Society of Engineering Education Annual Conference and Exposition
June 12-15, 2005, Portland Oregon

Interweaving Numerical Methods Techniques in Multi-Semester Projects

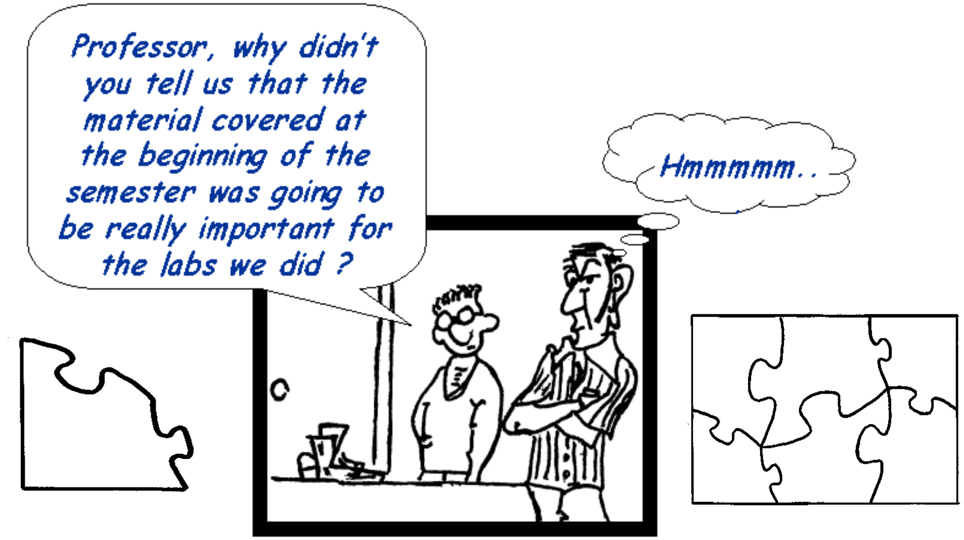


Dr. Peter Avitabile, Dr. John McKelliget, Tracy Van Zandt
Mechanical Engineering Department
University of Massachusetts Lowell

Problems teaching mechanical engineering

Students have difficulties because:

- Do not understand need for basic STEM material
- Course material appears disjointed

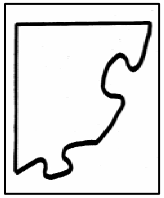


Student views material in a disjointed fashion

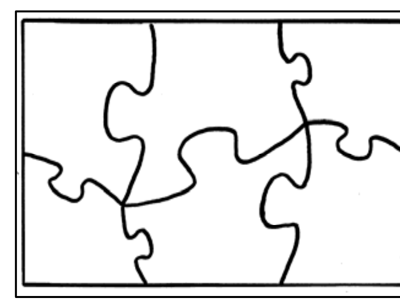
Professor clearly sees how pieces fit together

- Modular course environment reinforces this disjointed appearance
- Students hit the RESET button after each course

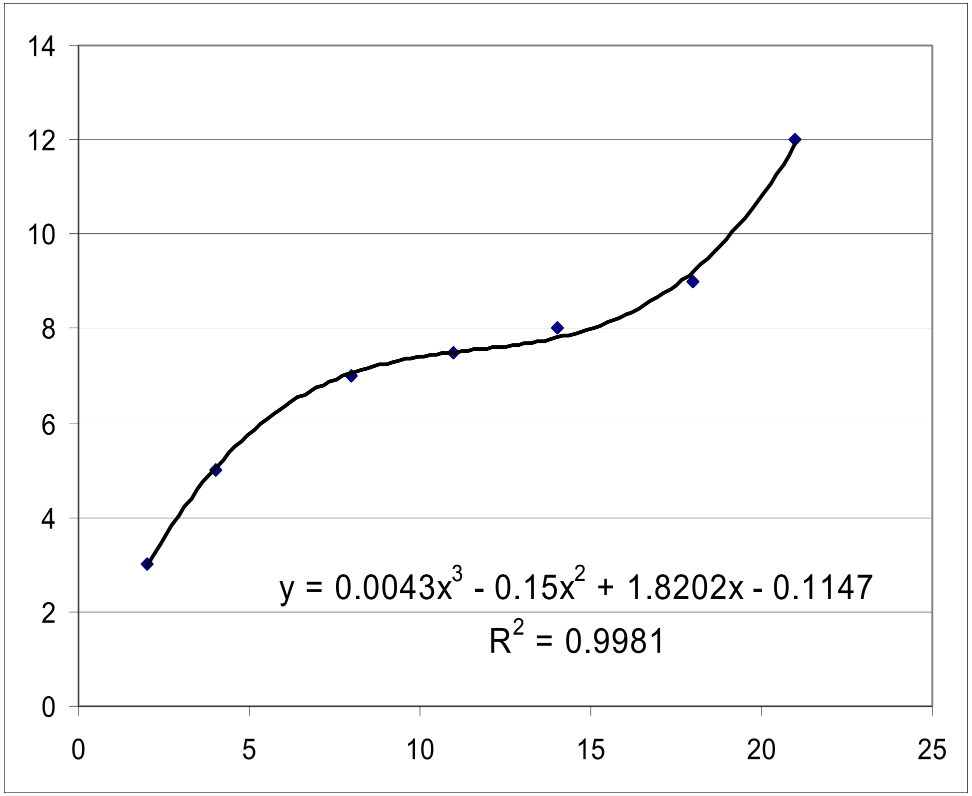
Problems teaching mechanical engineering



- Students take Math Methods in Junior year
- Learn numerical integration and differentiation and regression in preparation for lab
- Two projects have been added to this course to go beyond the typical textbook presentations of these topics
- Students can then use MATLAB GUIs to further their understanding



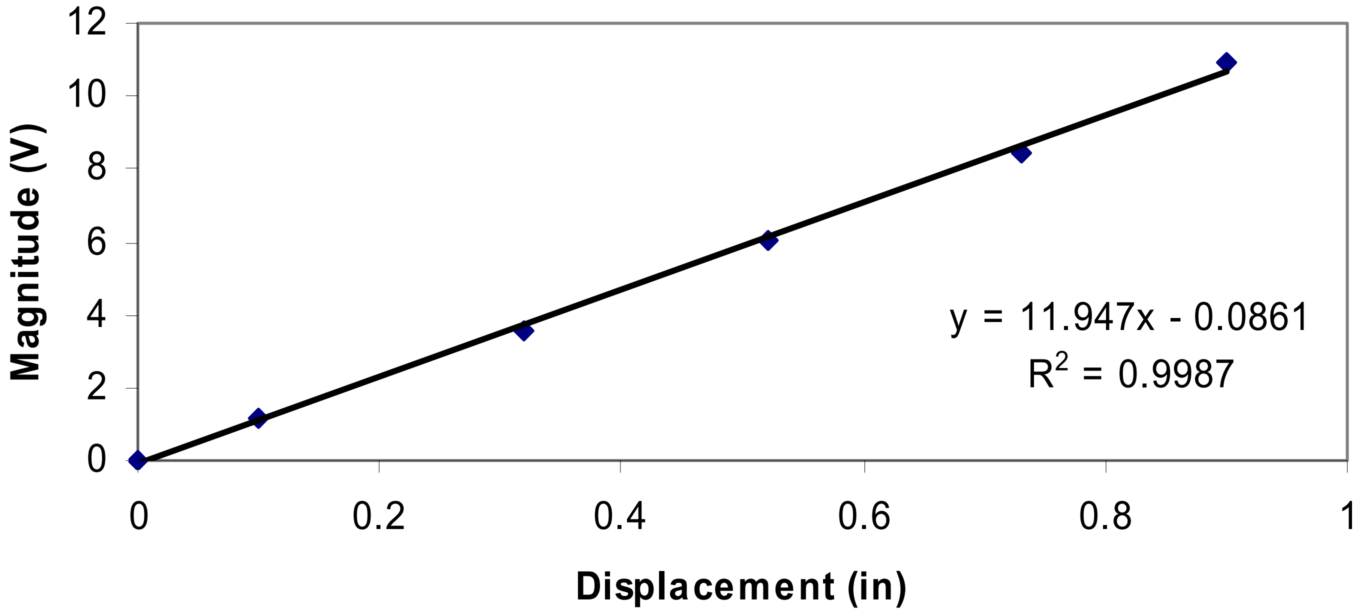
Regression analysis



Regression analysis

- Vital tool used widely in engineering analysis
- Traditional presentation:

  Students are given a simple data set
  Perform regression
  Report R^2 value & equation of line



Regression analysis

Problems with traditional approach:

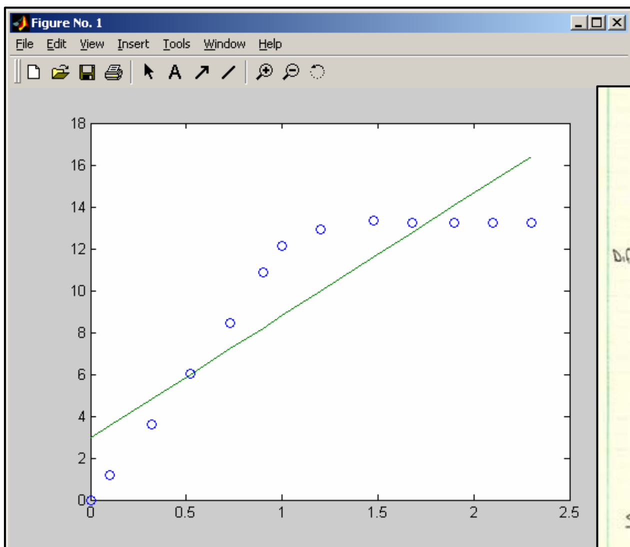
- Students simply apply regression over all data points
- Students think that increasing the order of regression always improves the result, because it improves the R^2 value

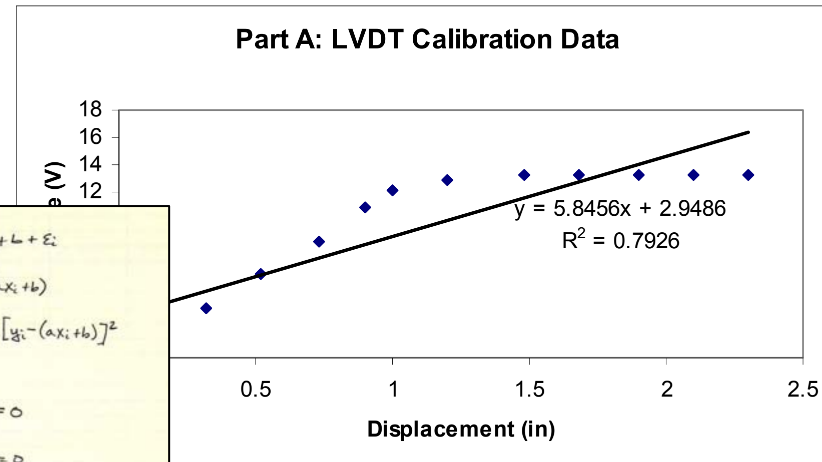
We want to make them THINK about how to best perform regression on their data

Regression analysis

Students perform regression using hand calculations, MATLAB, and/or Excel

Tutorial material is given





x	y
0	0
0.52	6.052
1	12.116
1.68	13.243

$y_i = mx_i + b + \varepsilon_i$

$\varepsilon_i = y_i - (ax_i + b)$

$\varepsilon_i^2 = z = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2$

Differentiate z with respect to a and b

$\frac{\partial z}{\partial a} = -2 \sum_{i=1}^{n} x_i [y_i - (ax_i + b)] = 0$

$\frac{\partial z}{\partial b} = -2 \sum_{i=1}^{n} [y_i - (ax_i + b)] = 0$

1) $a \sum_{i=1}^{n} x_i + bn = \sum_{i=1}^{n} y_i$

2) $a \sum_{i=1}^{n} x_i^2 + b \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$

$\Sigma x_i^2 = 4.09$
$\Sigma x_i = 3.2$
$\Sigma y_i = 31.411$
$\Sigma x_i y_i = 37.51$

Solve simultaneous equations to get:

$a = 8.085$

$b = 1.392$

$y = 8.085x + 1.392$

$R^2 = 1 - \frac{SSE}{SST}$ $\quad SSE = \sum_{i=1}^{n} (y_i - f(x_i))^2$

$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$

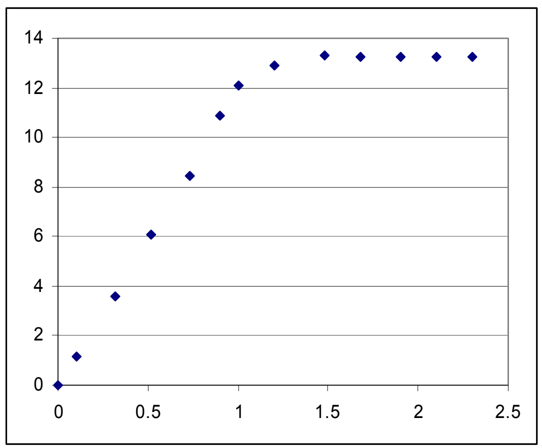
$R^2 = 0.879$

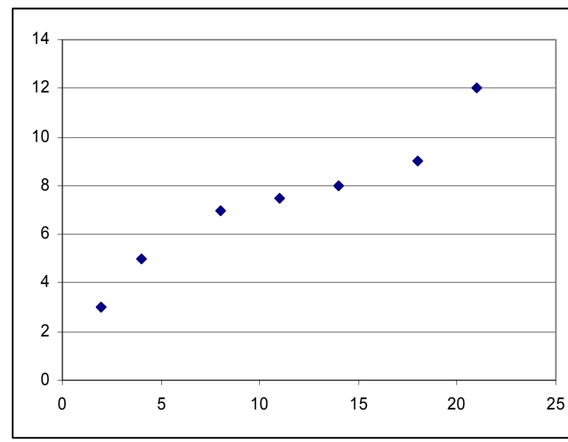
Regression analysis

Two major points we'd like them to consider:

1. **Should the regression be performed over the whole range of the data?**
2. **What order of regression is most appropriate?**

Students are given two data sets to study

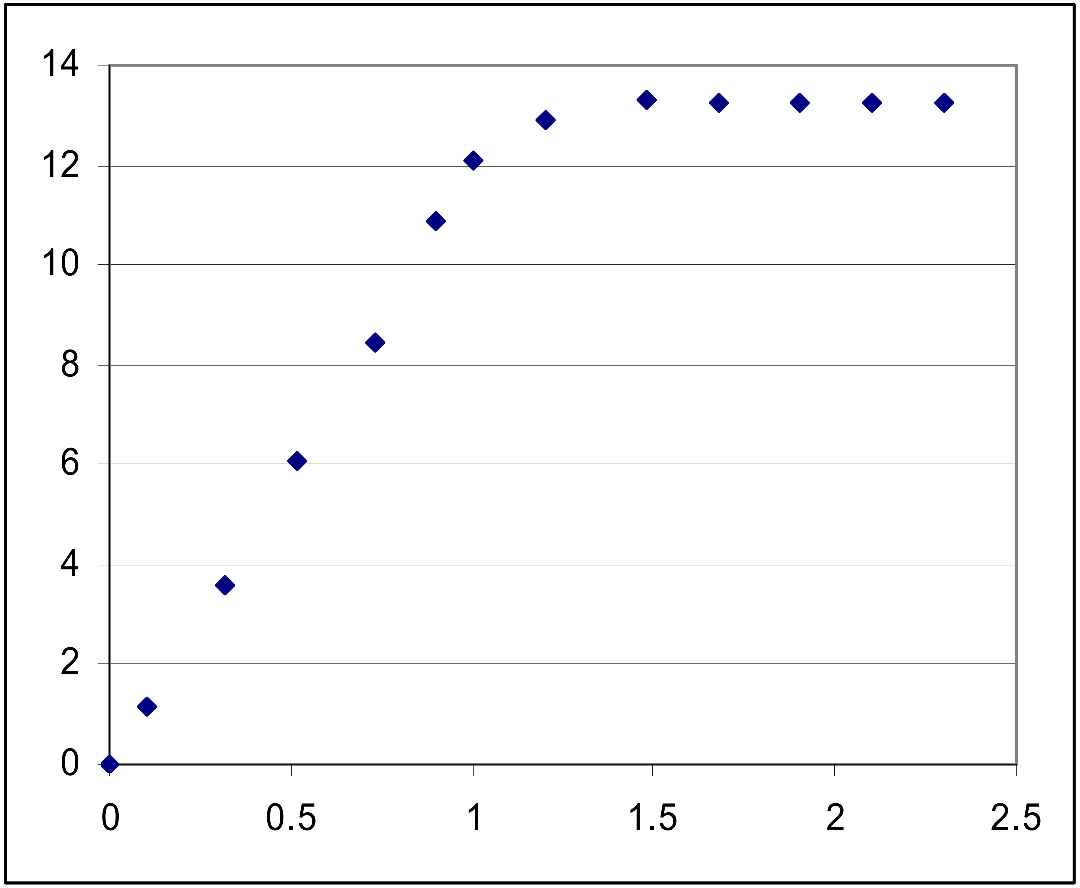




Regression analysis

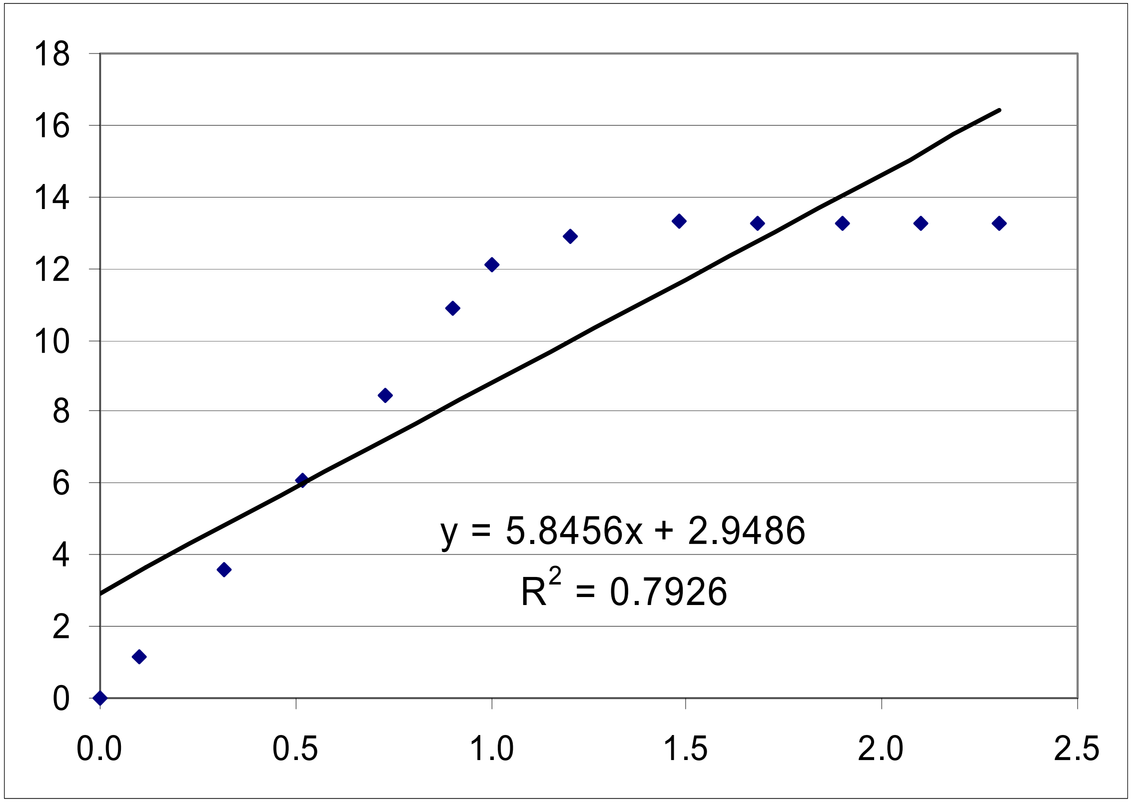
Data set #1

- Bi-Linear
- Calibration data from an LVDT

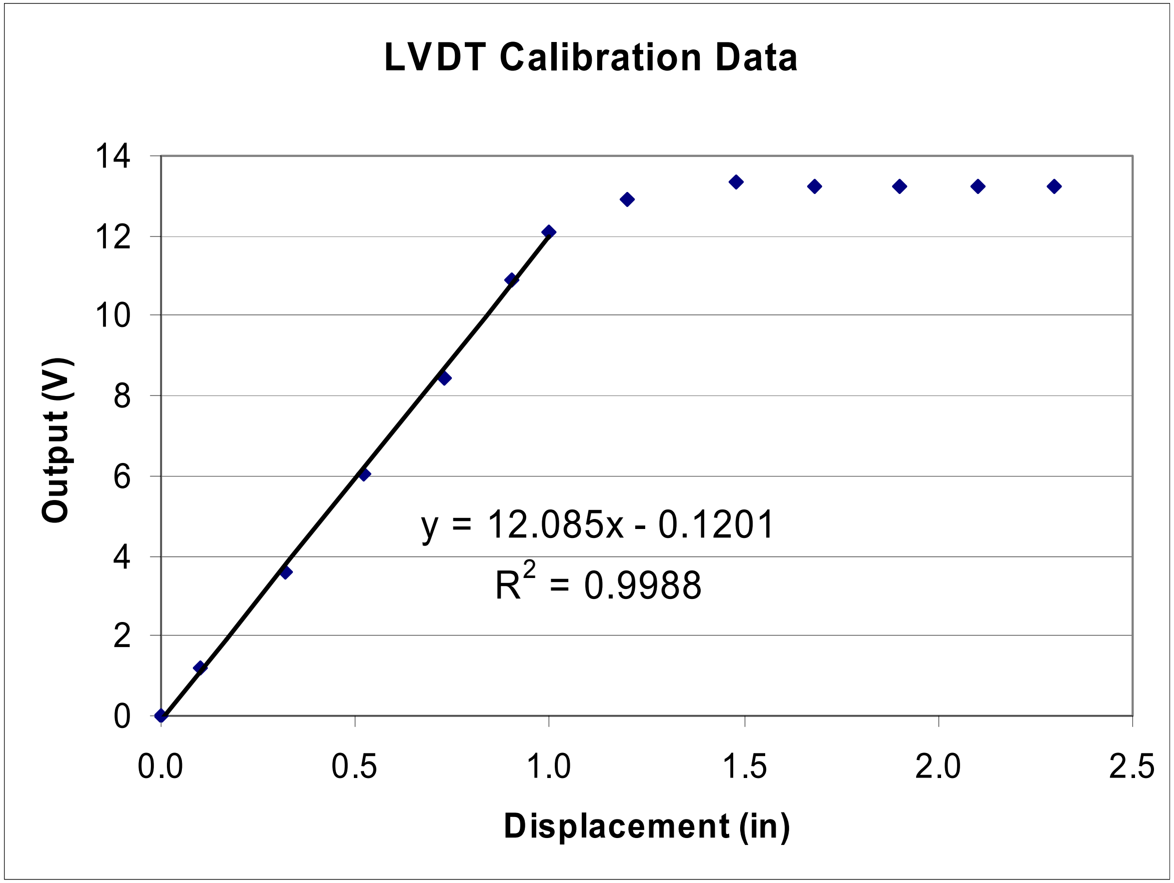


Regression analysis

Fit regression line through all data points



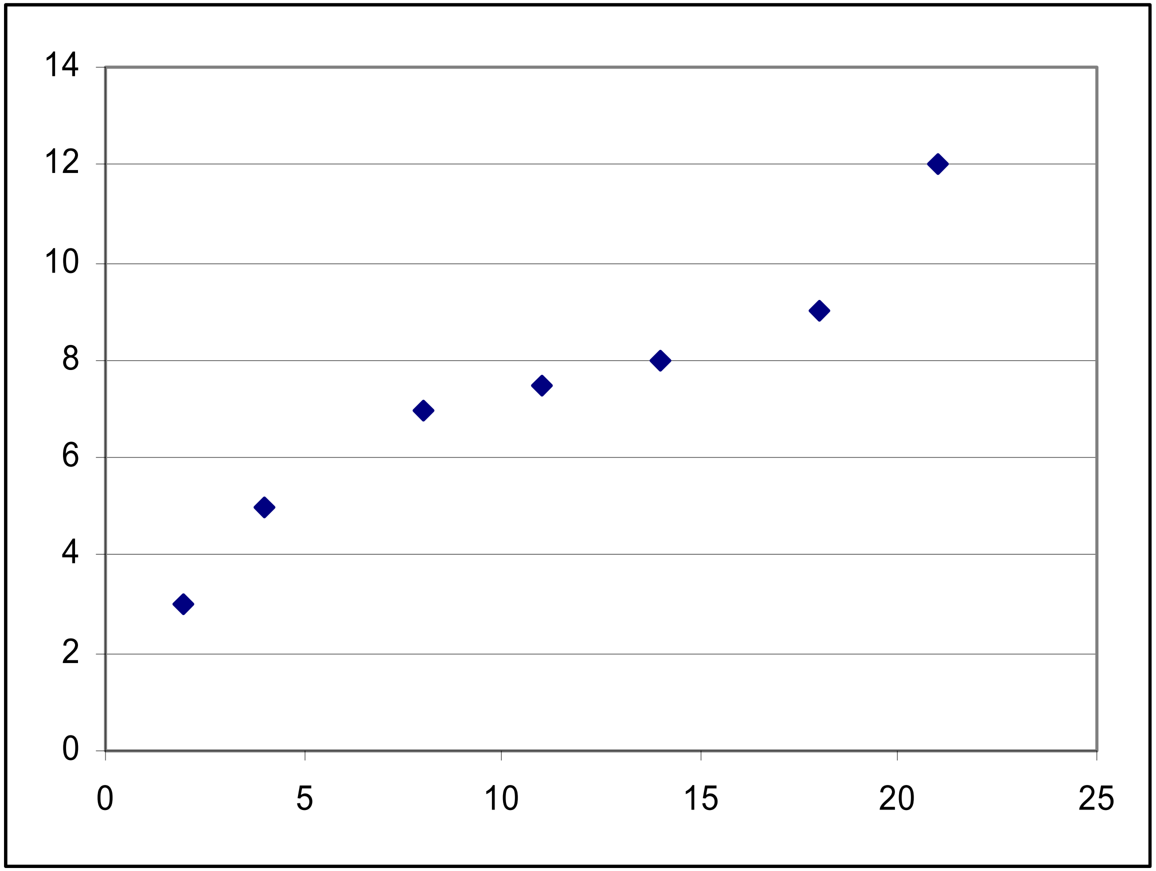
But an LVDT is linear only over a certain range



Regression analysis

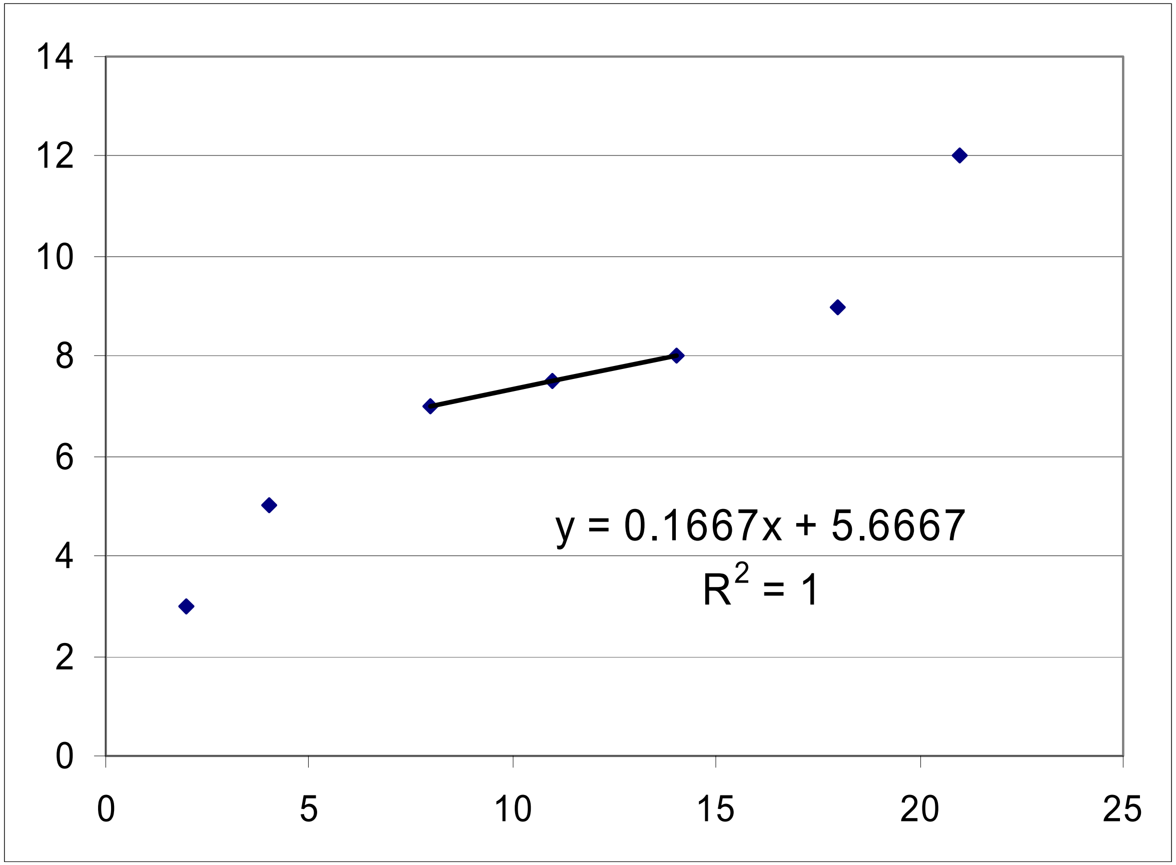
Data set #2

- *Appears cubic*
- *Data can be fit in multiple ways*

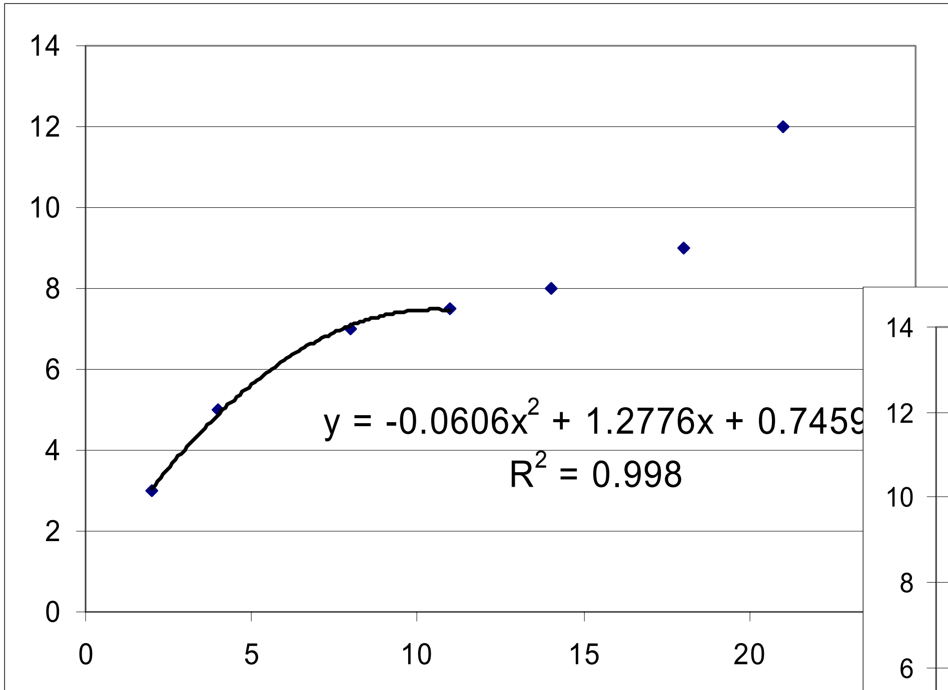


Regression analysis

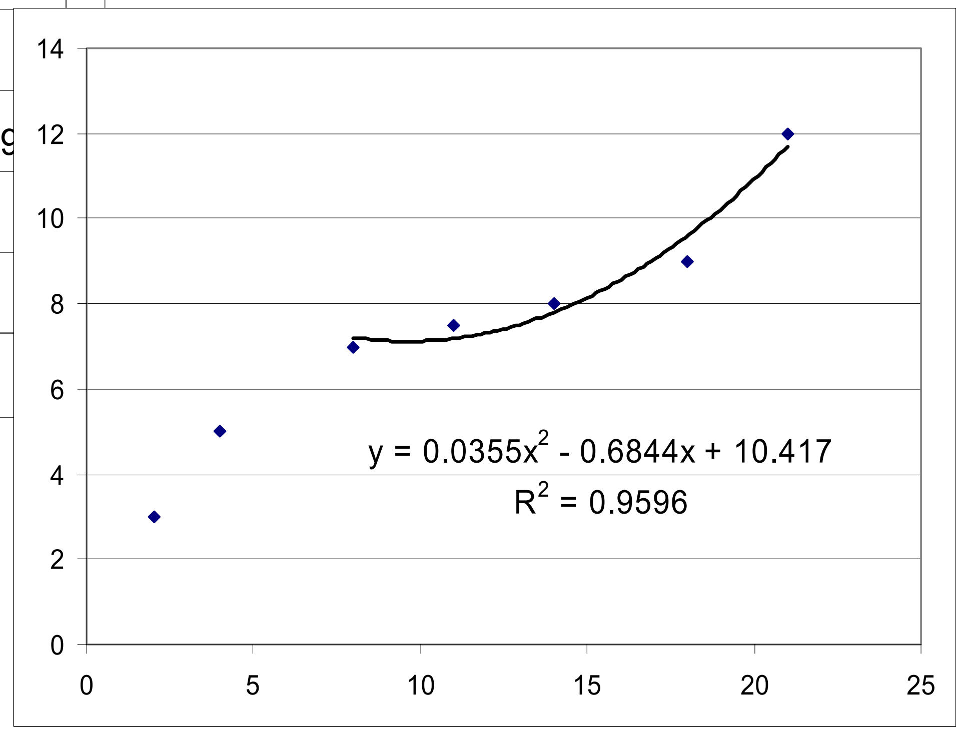
Central portion of data is linear



Regression analysis

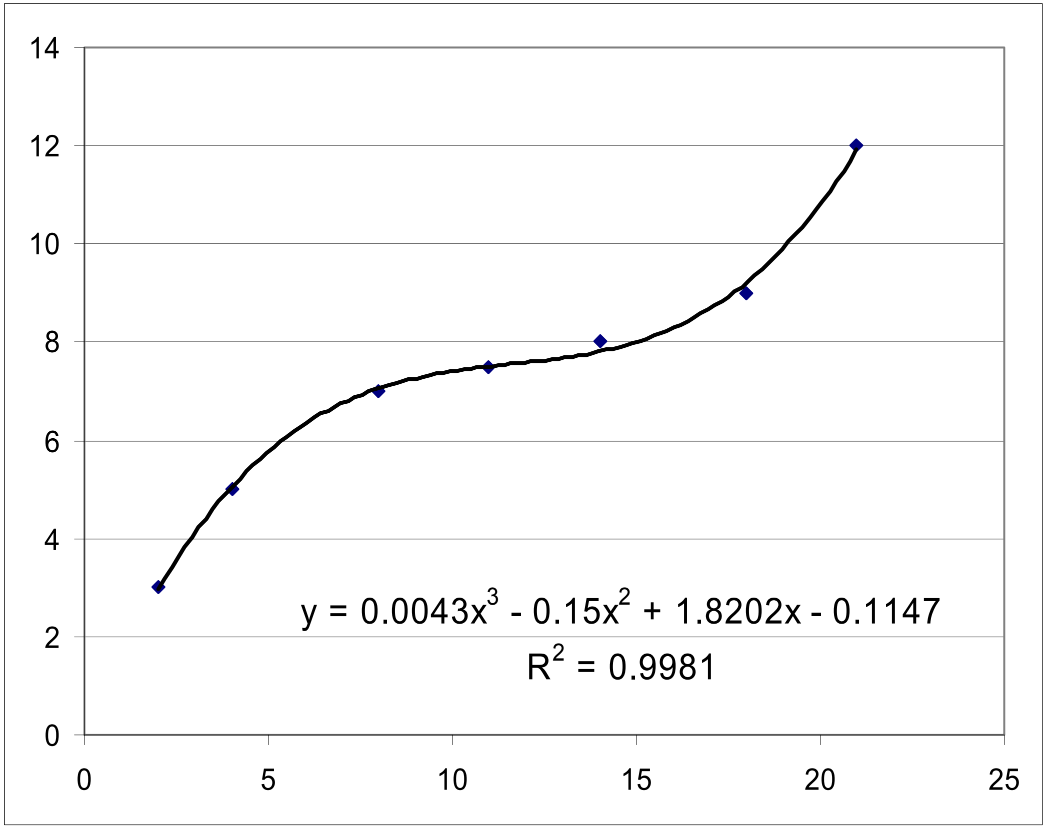


Upper and lower portions of data are somewhat parabolic



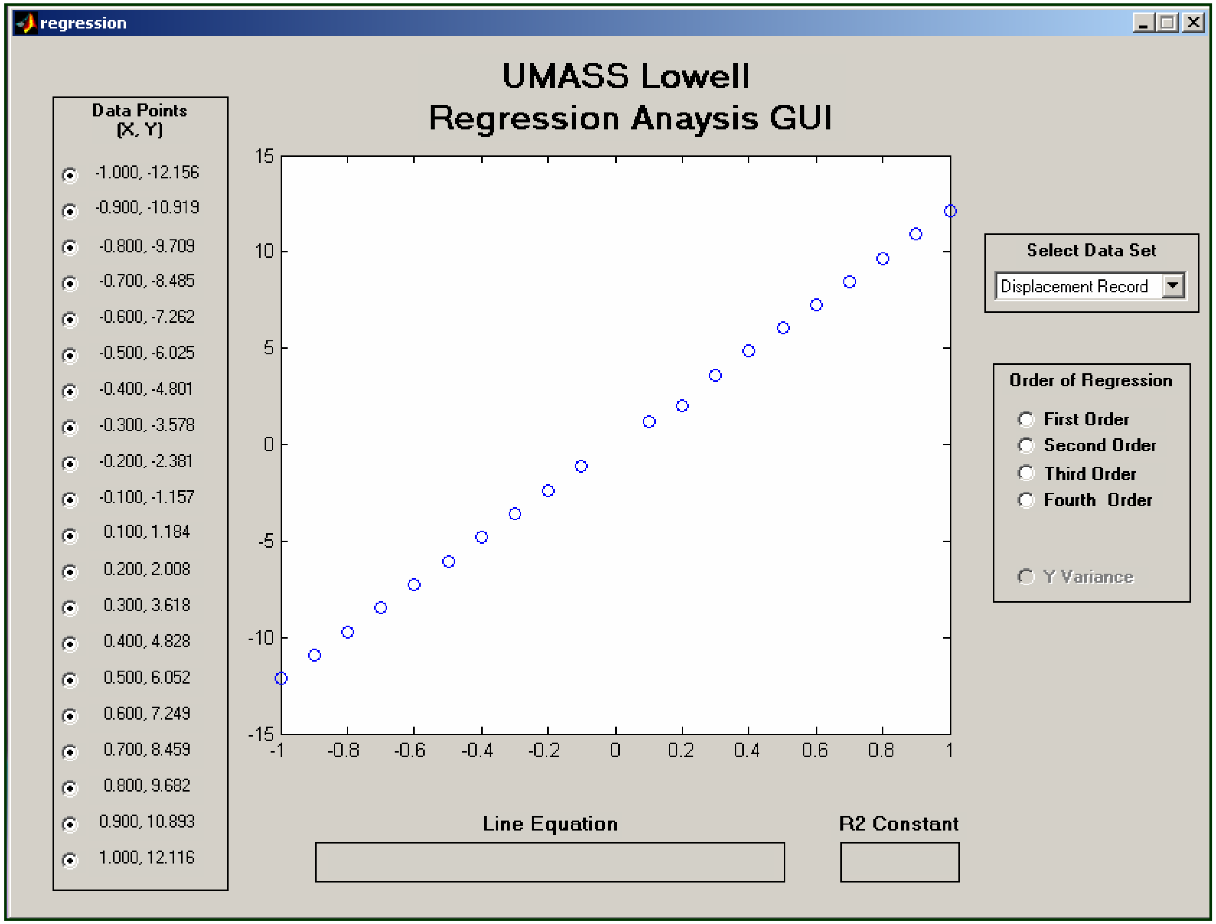
Regression analysis

Over entire range, data is cubic

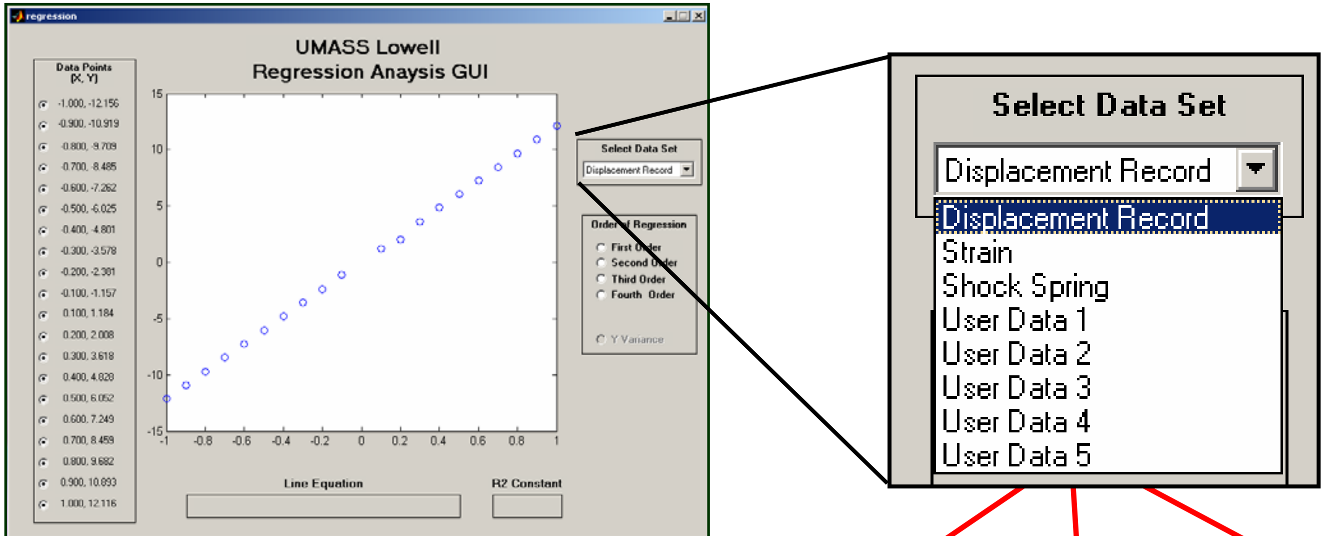


Regression analysis

Students can then use Matlab GUI to further explore regression

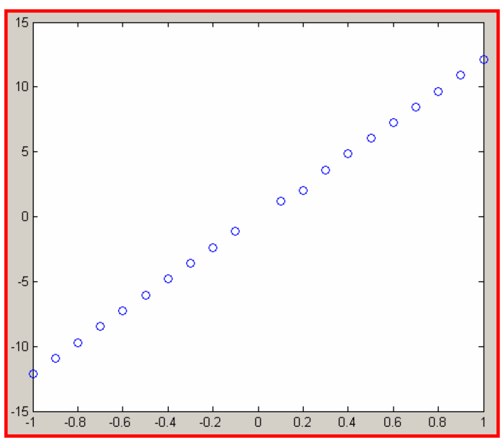


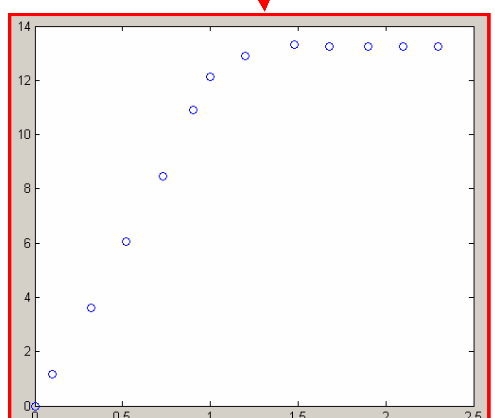
Regression analysis

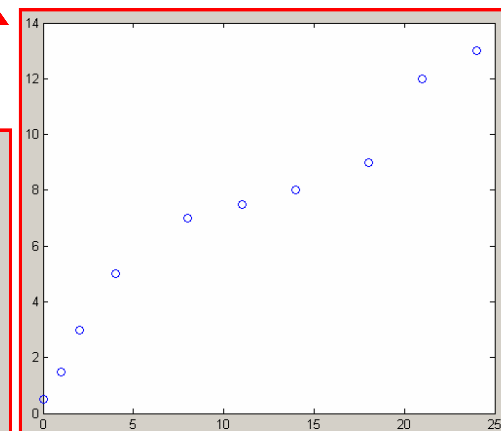


Multiple data sets available

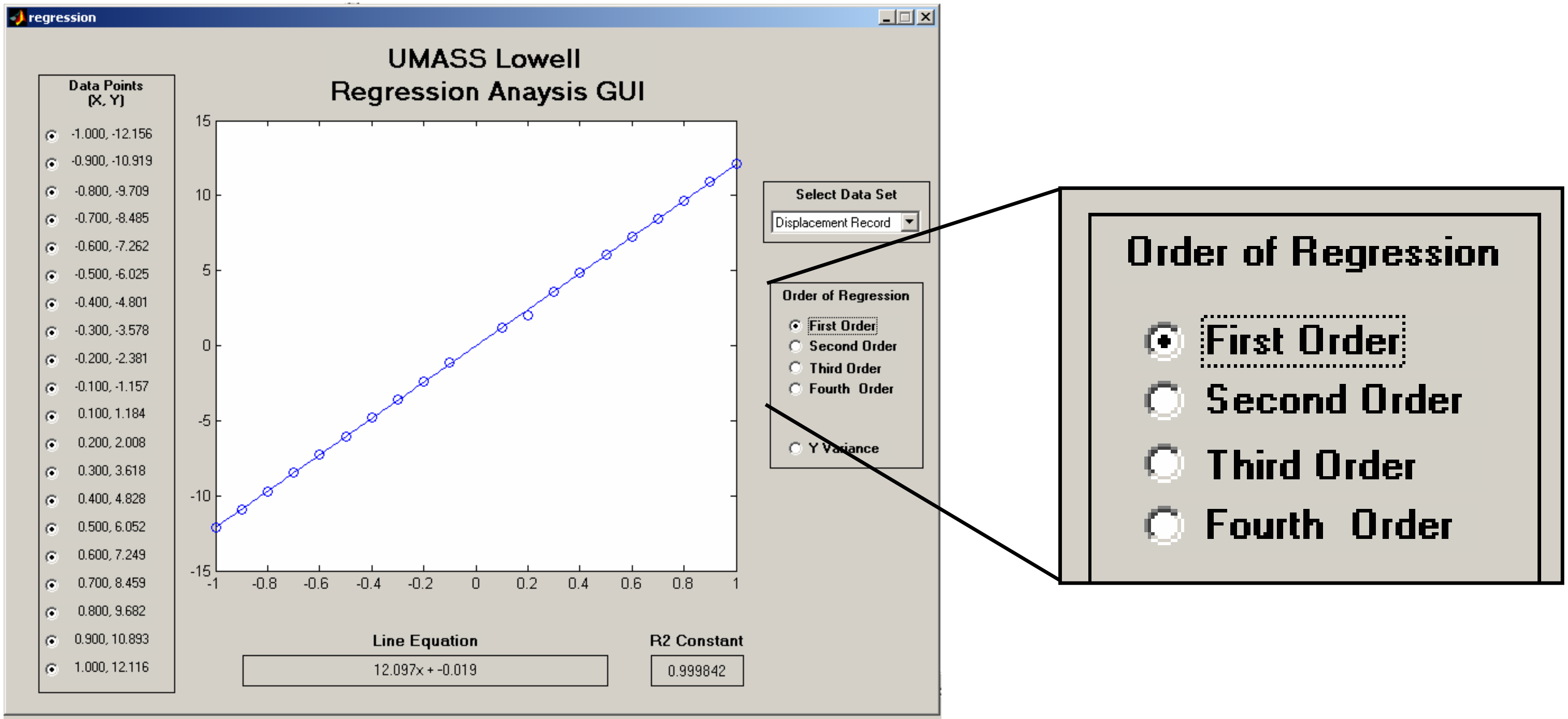
Or user can input data set





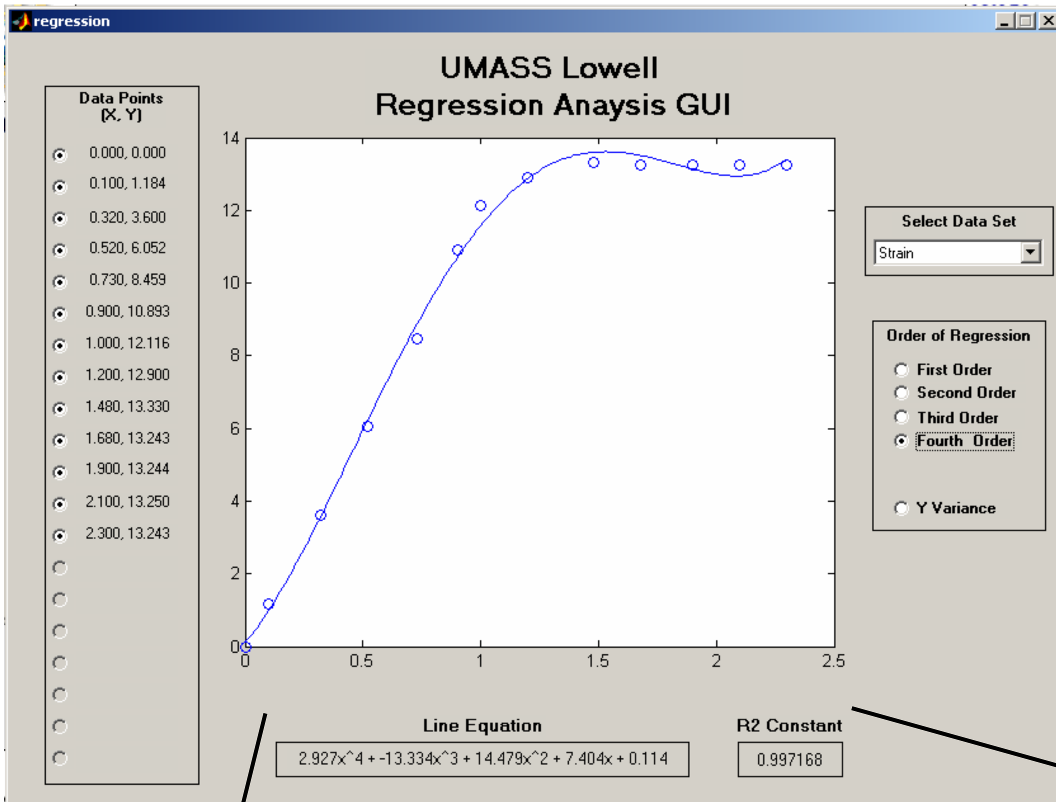


Regression analysis

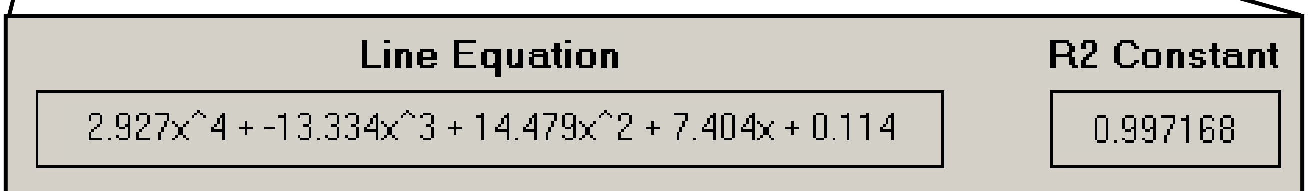


Different order regressions may be performed

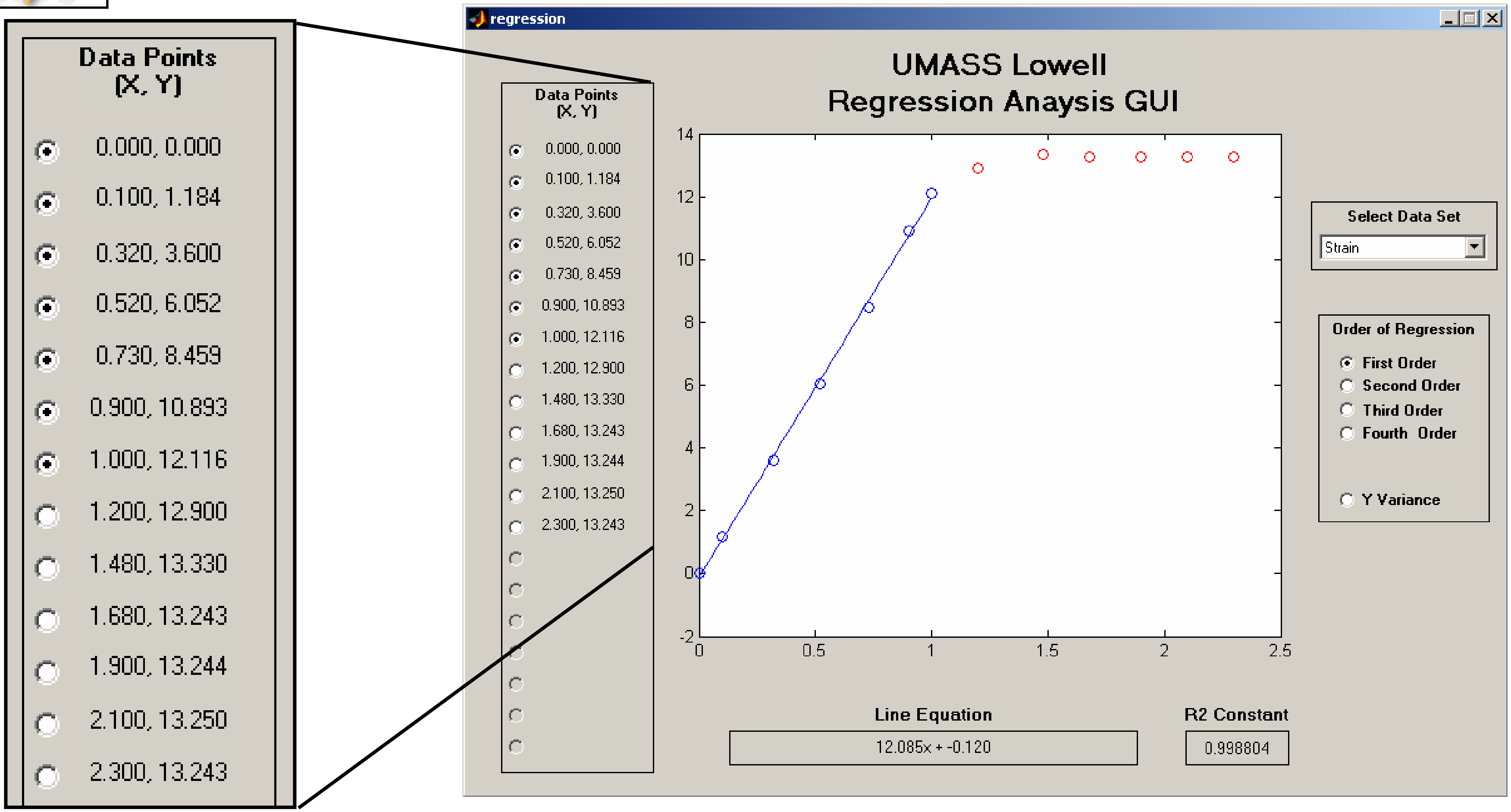
Regression analysis



The equation of the line and the R^2 value are reported

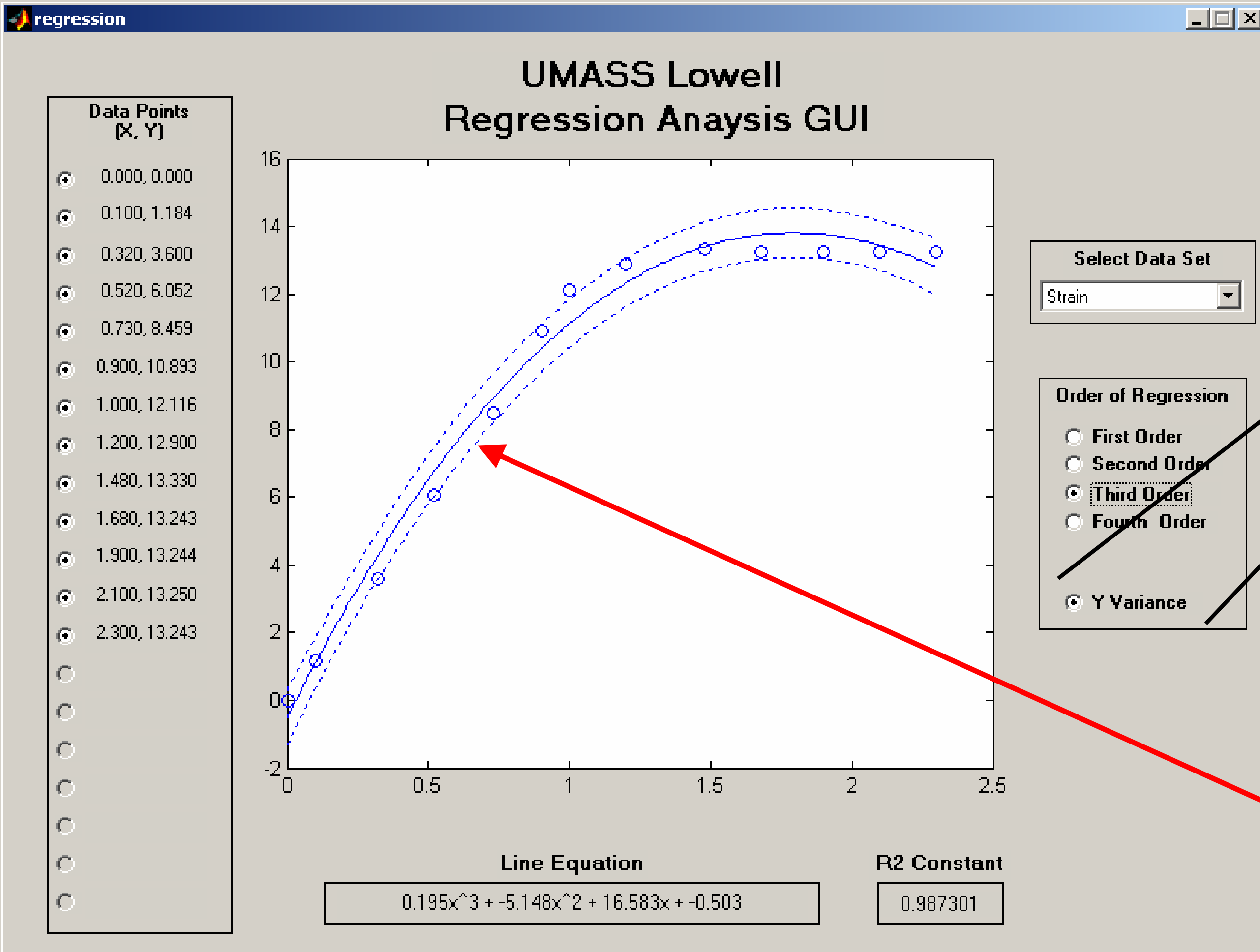


Regression analysis

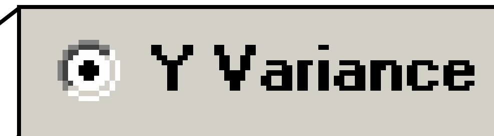


Points may be deselected, and they will not be included in the regression calculation

Regression analysis

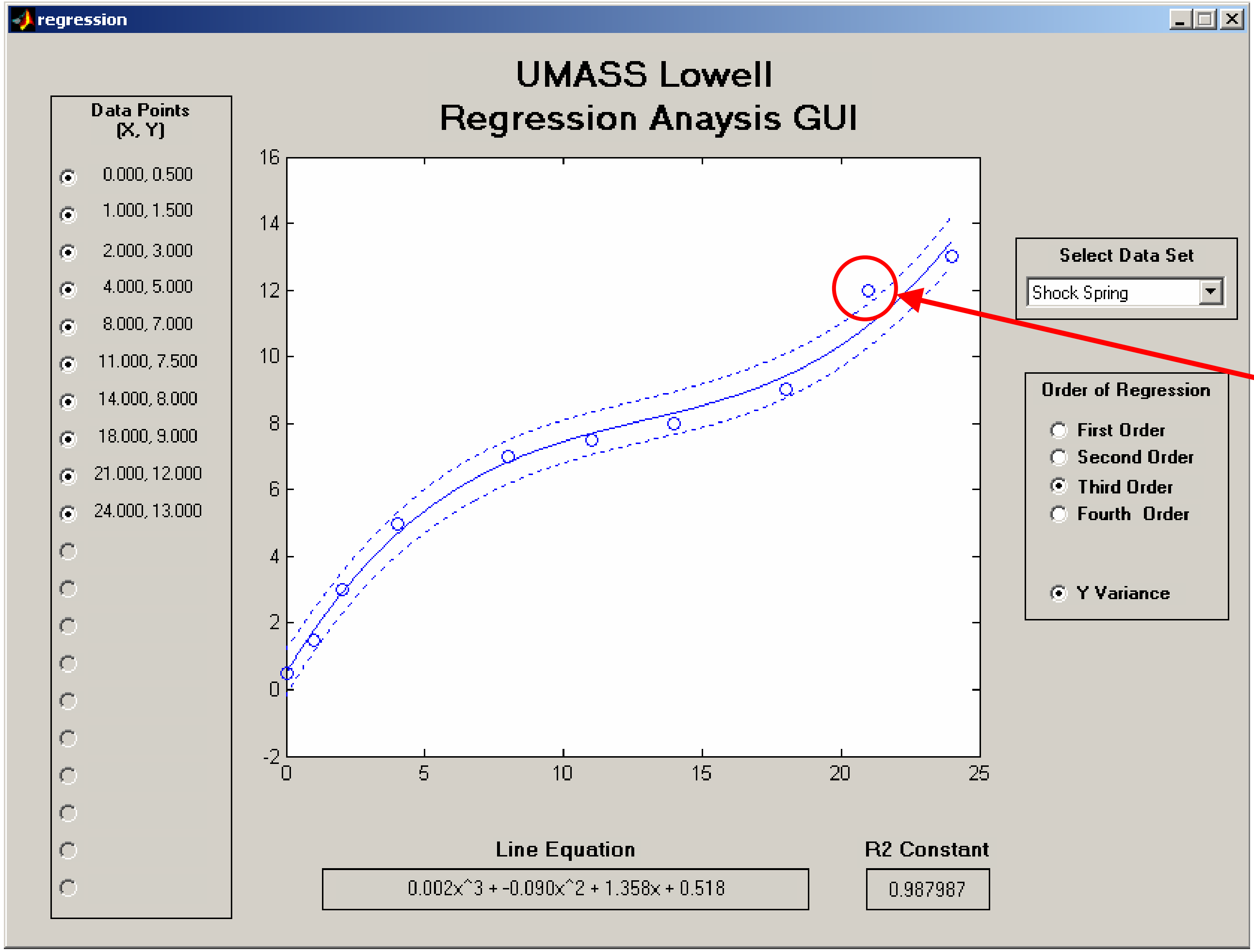


The GUI can also plot the y-variance of the data



Lines are drawn above and below the regression line at a distance of 3 σ

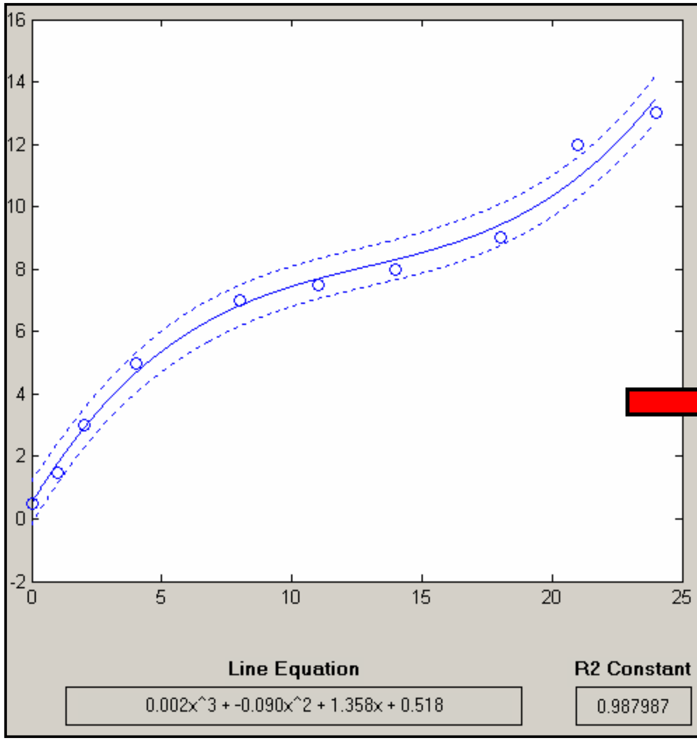
Regression analysis

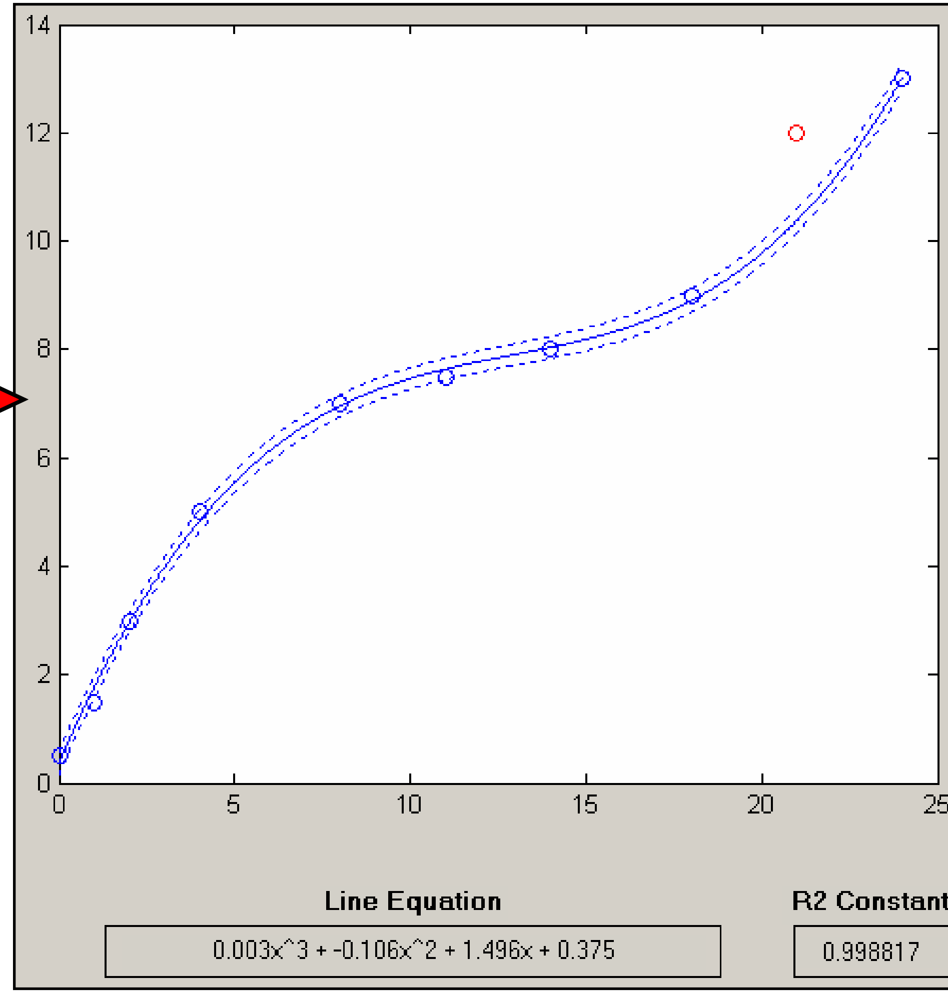


Therefore, if a data point lies outside the 3 σ range,

either the data point or the regression line should probably be questioned

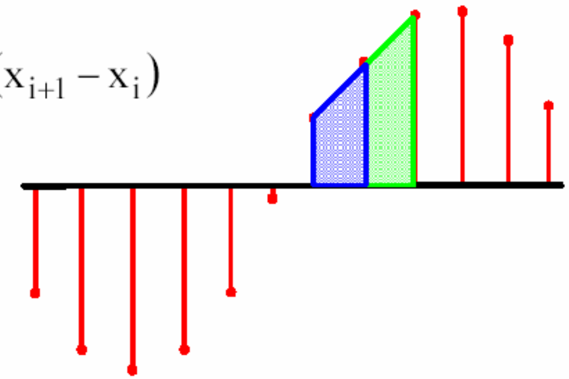
Regression analysis



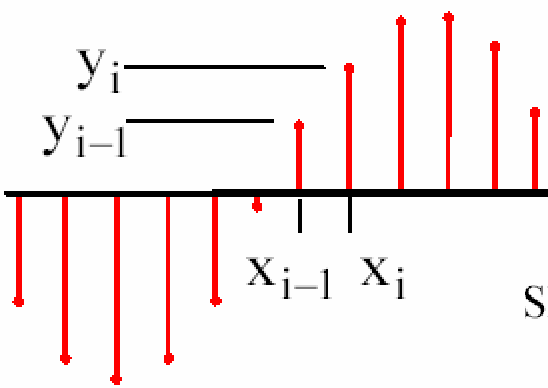


In this case, ignoring one data point greatly improves the regression

$$I_i = I_{i-1} + \frac{y_{i+1} + y_i}{2}(x_{i+1} - x_i)$$



Numerical differentiation and integration



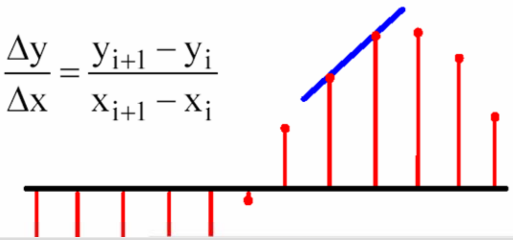
$$\text{SLOPE} = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{y_{i+1} - y_{i-1}}{x_{i+1} - x_{i-1}}$$

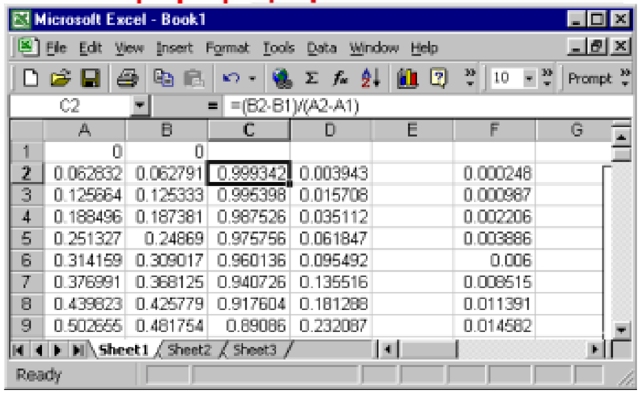
Numerical differentiation and integration

Traditional teaching method:

Students use different differentiation and integration tools on simple, well-behaved analytical data sets

$$\text{SLOPE} = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$





	A	B	C	D	E	F	G
1	0	0					
2	0.062832	0.062791	0.999342	0.003943		0.000248	
3	0.125664	0.125333	0.995398	0.015708		0.000987	
4	0.188496	0.187381	0.987526	0.035112		0.002206	
5	0.251327	0.24869	0.975756	0.061847		0.003886	
6	0.314159	0.309017	0.960136	0.095492		0.006	
7	0.376991	0.368125	0.940726	0.135516		0.008515	
8	0.439823	0.425779	0.917604	0.181288		0.011391	
9	0.502655	0.481754	0.89086	0.232087		0.014582	

Problem with traditional method:

Students get to the lab course, and don't understand how to handle real data

Numerical differentiation and integration

Concepts that students have trouble with:

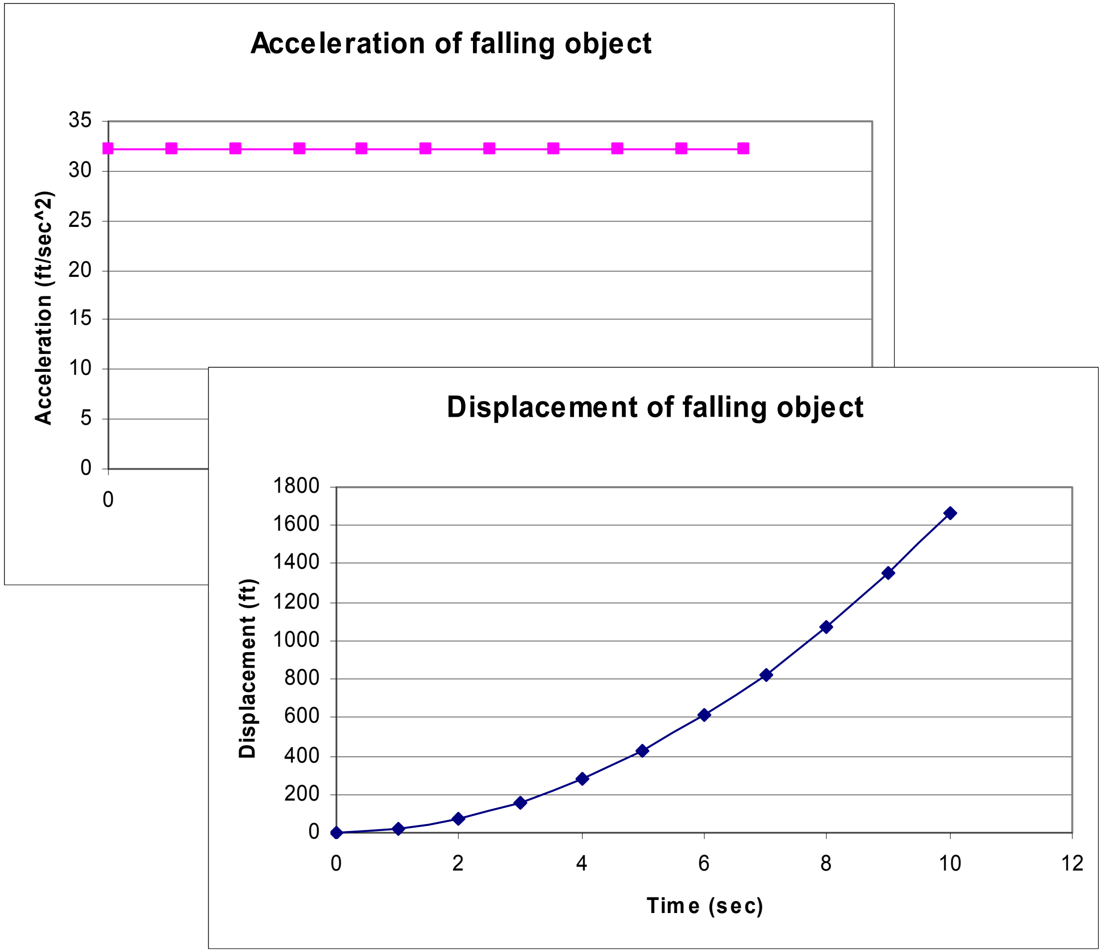
- **Understanding the use of initial conditions in integration**
- **Processing data that has noise, bias, drift**
- **Processing data with large time steps**

This project forces the students to consider these issues when they first see the material, so they will be better prepared when they see these problems in lab

Numerical differentiation and integration

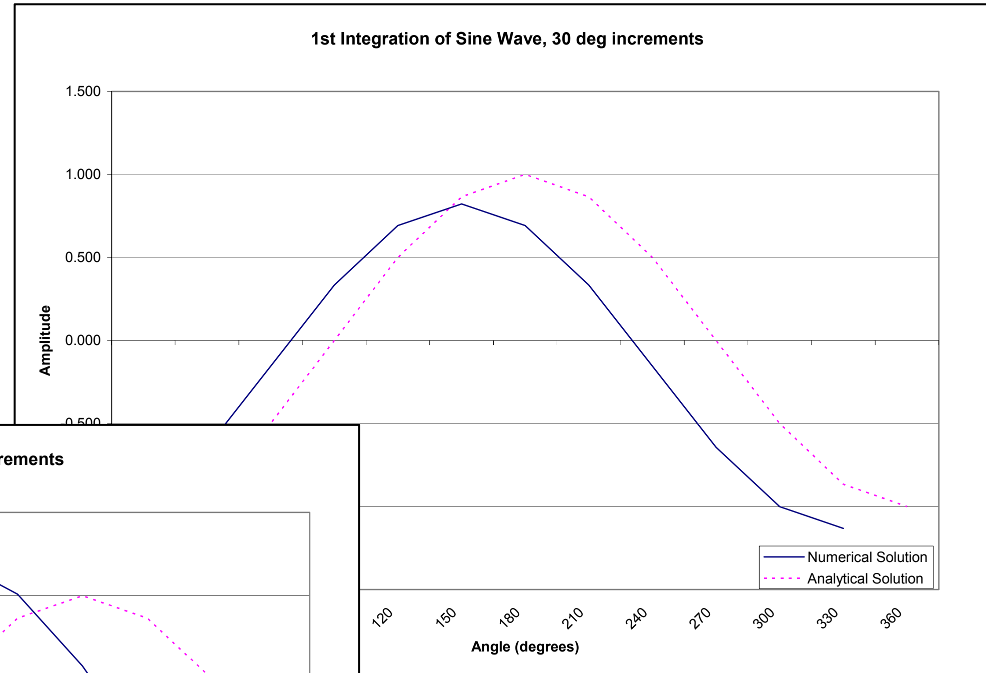
Students are given the displacement and acceleration of a falling object

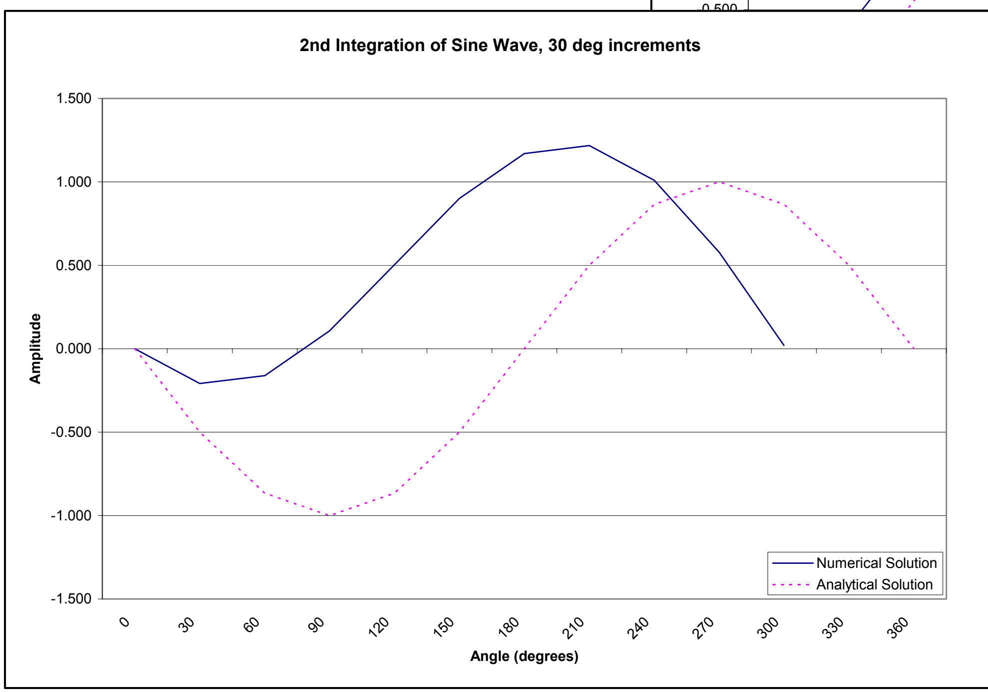
Object has initial velocity – students must find this I.C. by differentiating the displacement



Numerical differentiation and integration

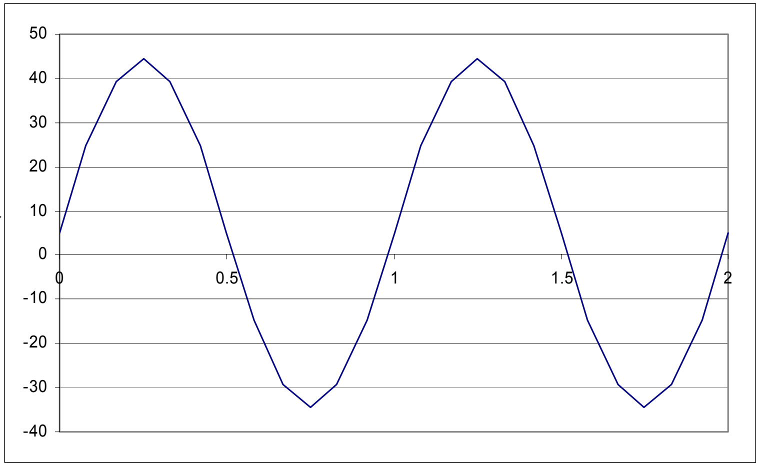
Students given data with too-large ΔT spacing



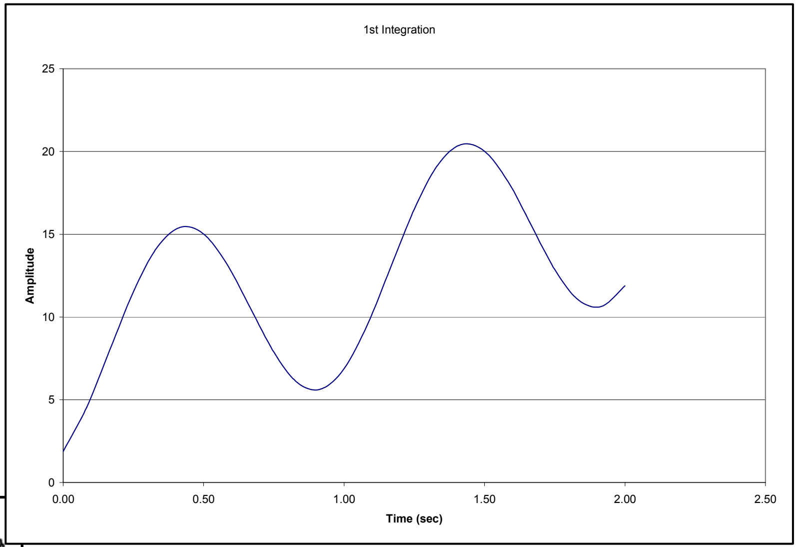


Numerical differentiation and integration

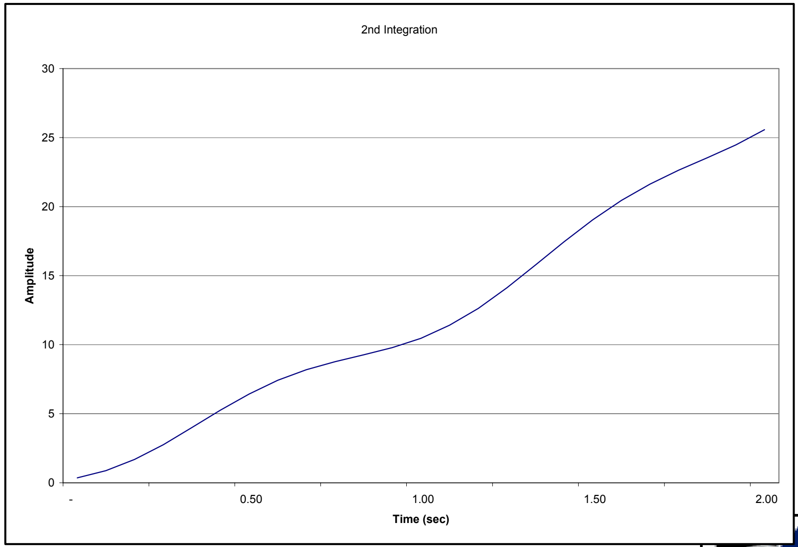
Students given data with small bias added

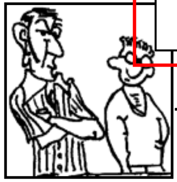


First integration



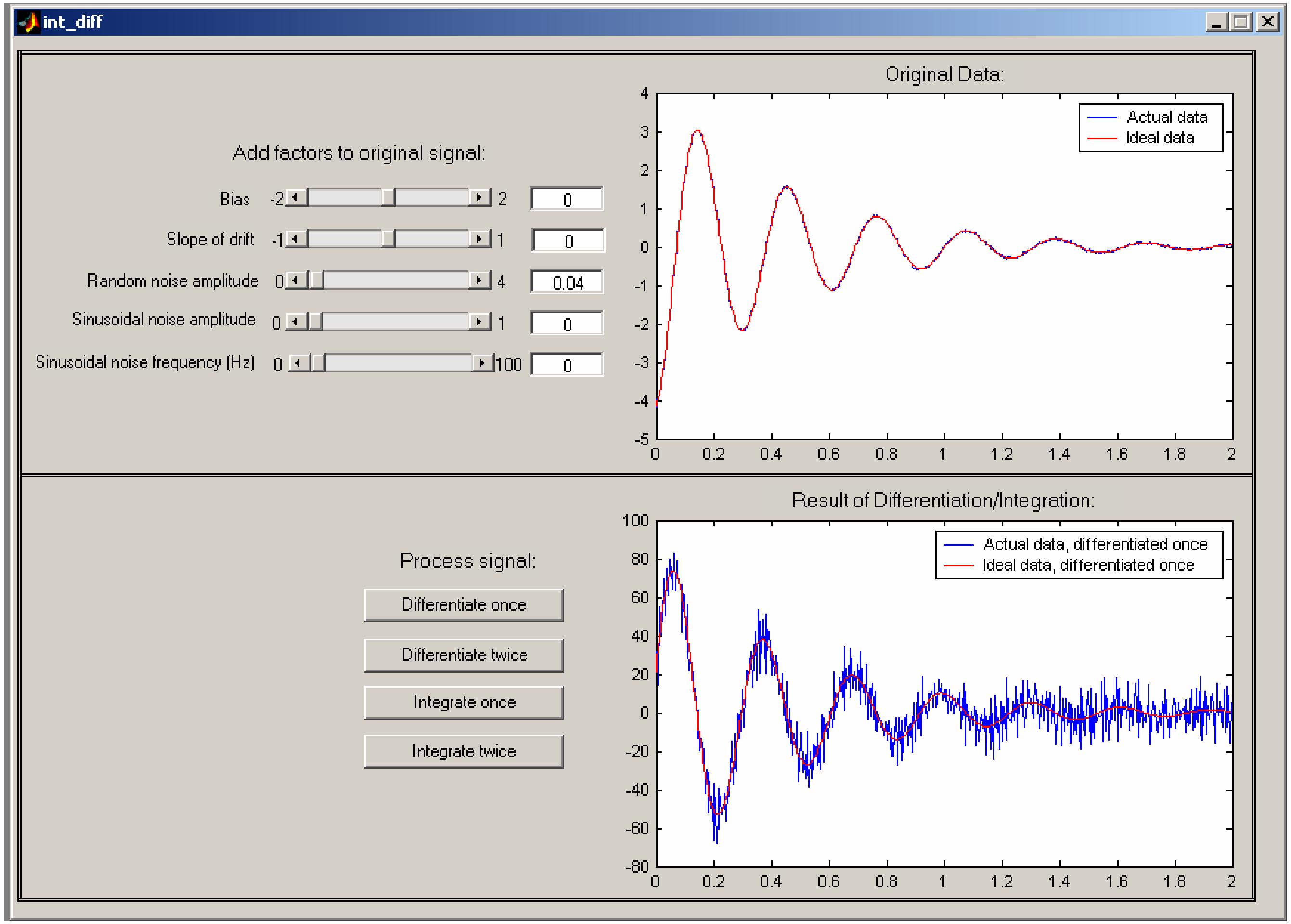
Second integration



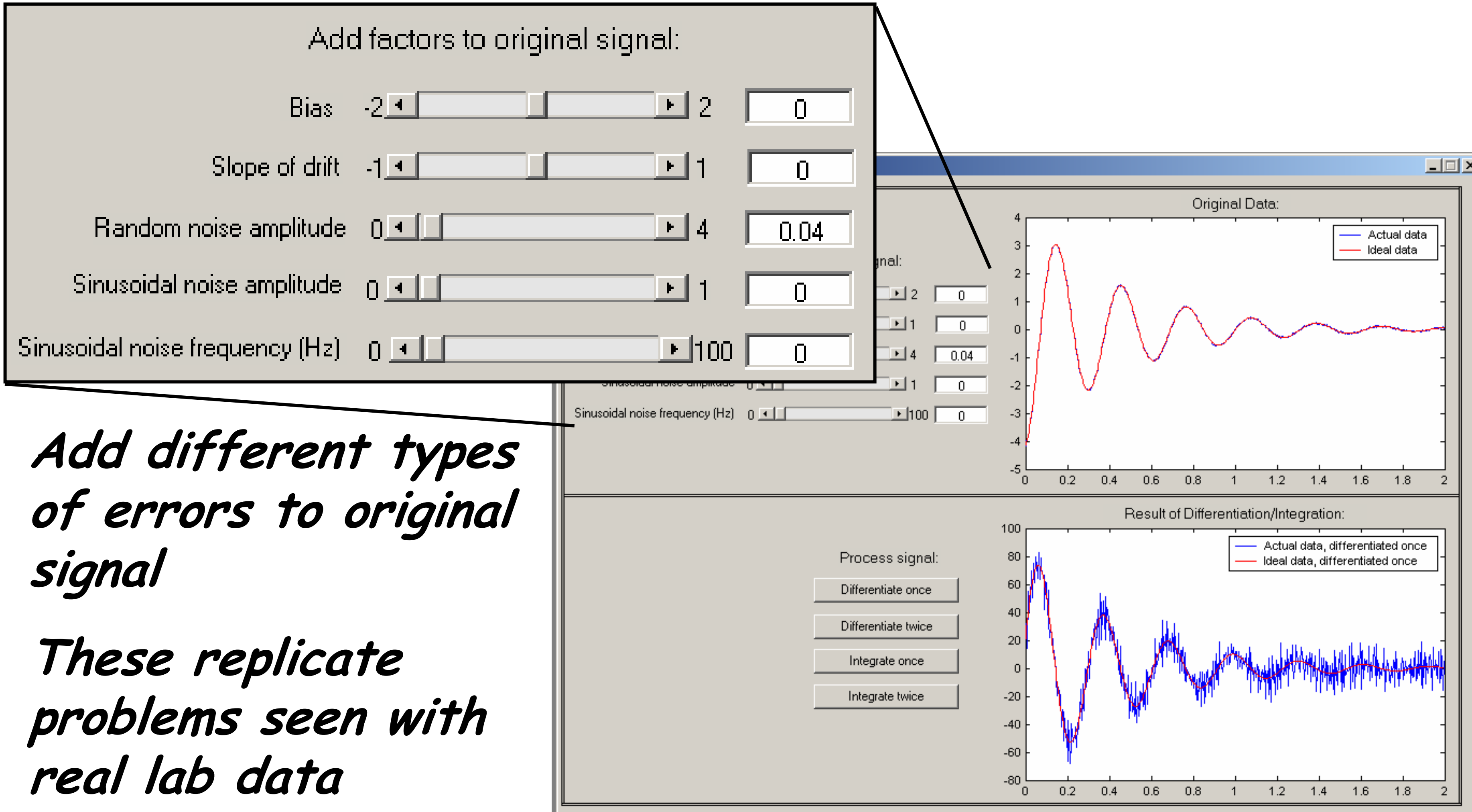


Numerical differentiation and integration

Students can then use the GUI to further explore numerical integration & differentiation



Numerical differentiation and integration

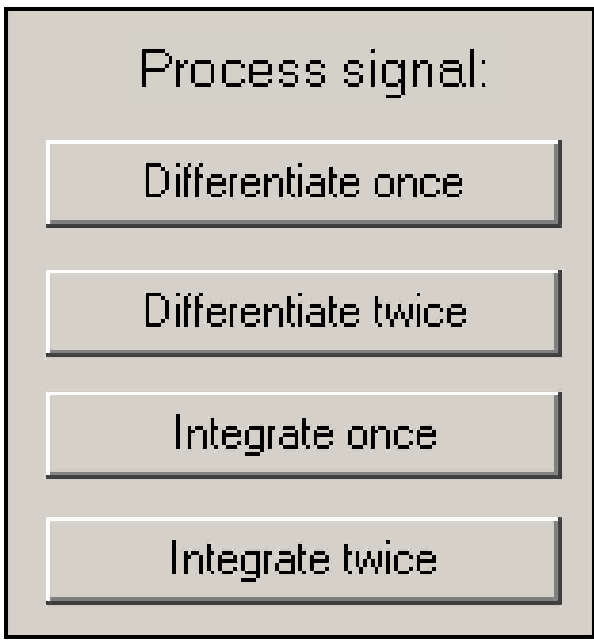


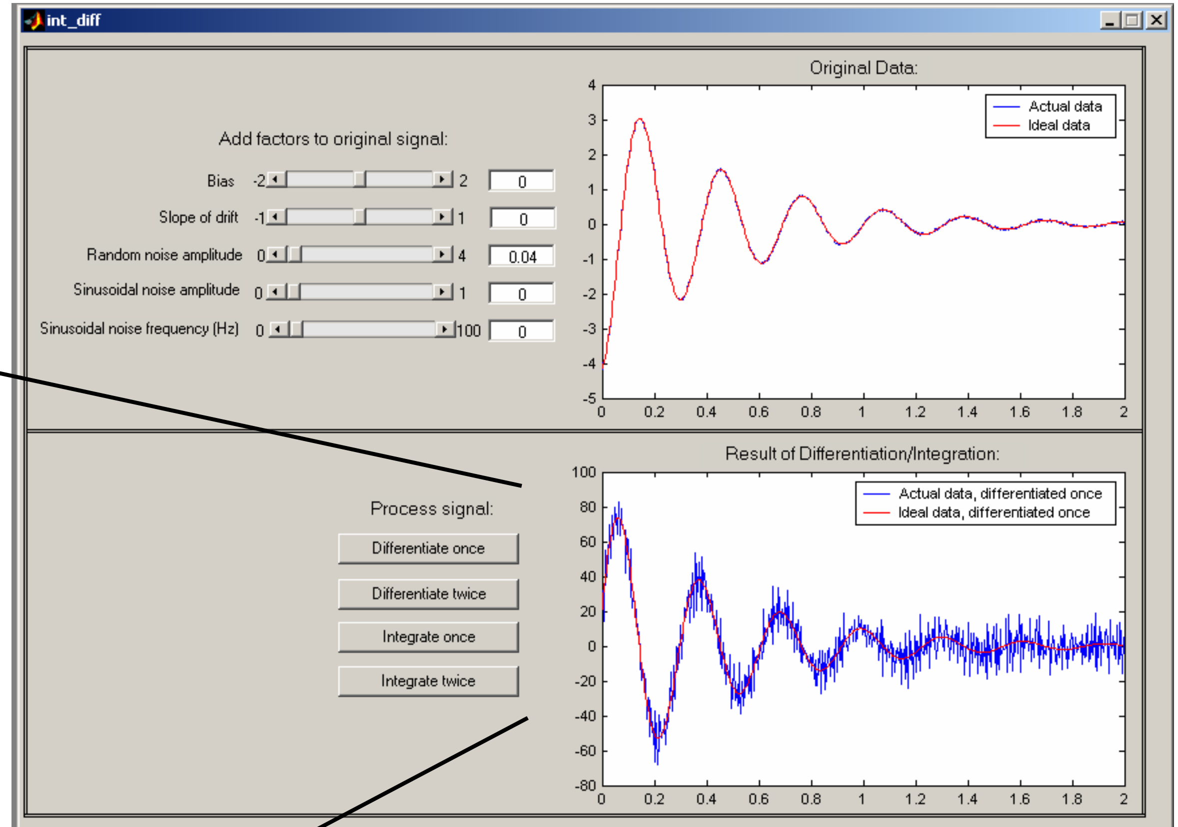
Add different types of errors to original signal

These replicate problems seen with real lab data

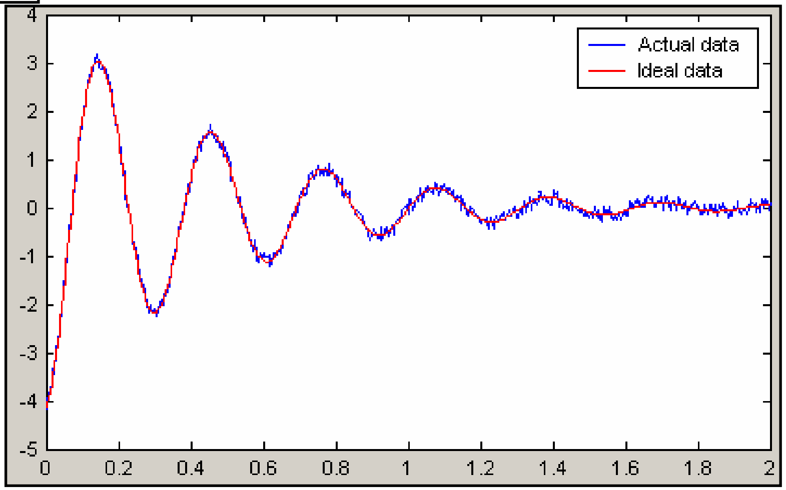
Numerical differentiation and integration

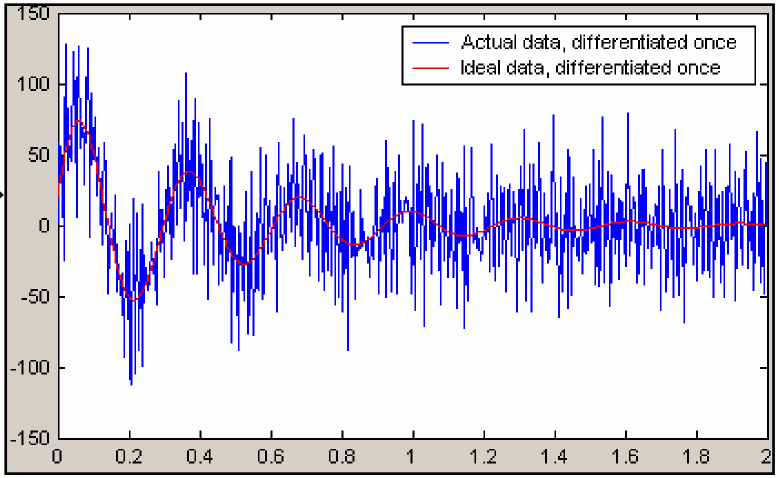
Differentiate and integrate signal

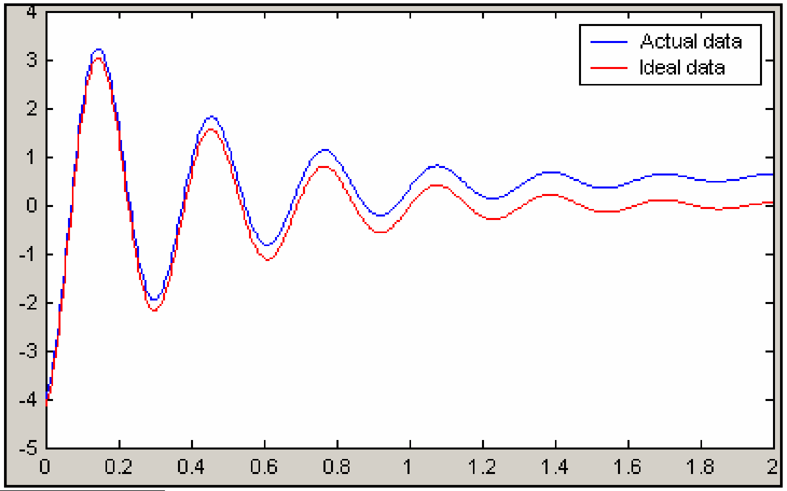


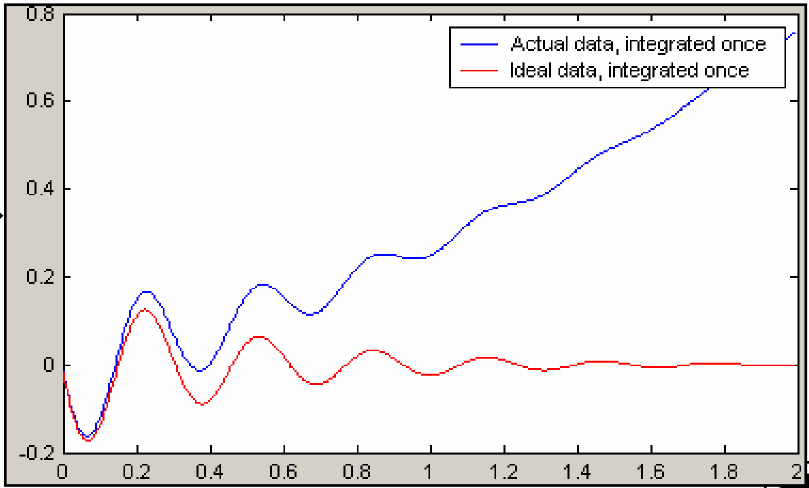


Numerical differentiation and integration





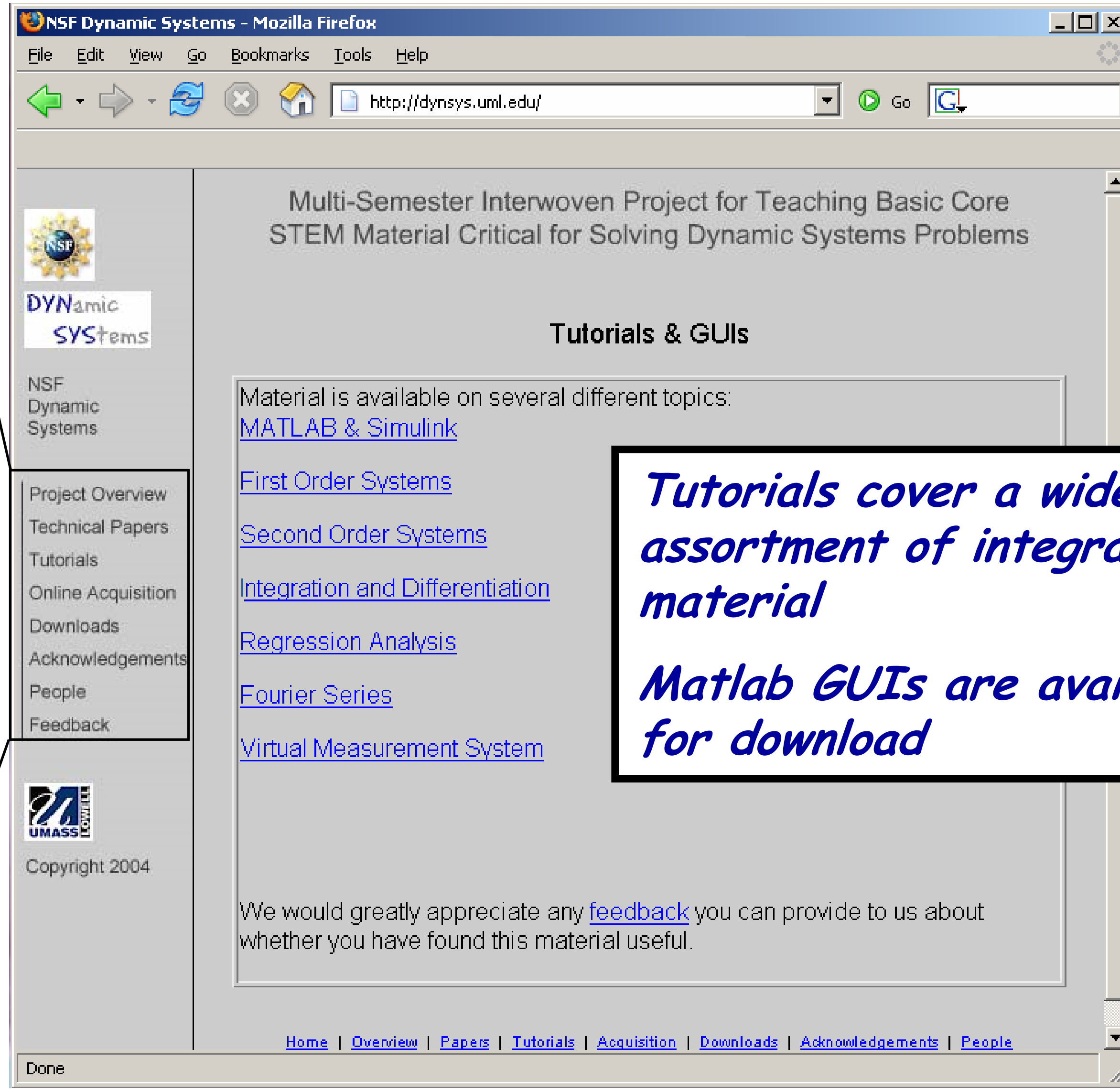




Conclusions

- Students have trouble retaining basic theory
- Projects can help them better understand the need for the material
- Two projects were developed to help them better understand numerical integration and differentiation and regression
- MATLAB GUIs are available to help them further explore these topics as they relate to real measurements

Webpage http://dynsys.uml.edu



Project Overview

Technical Papers

Tutorials

Online Acquisition

Downloads

Acknowledgements

People

Feedback

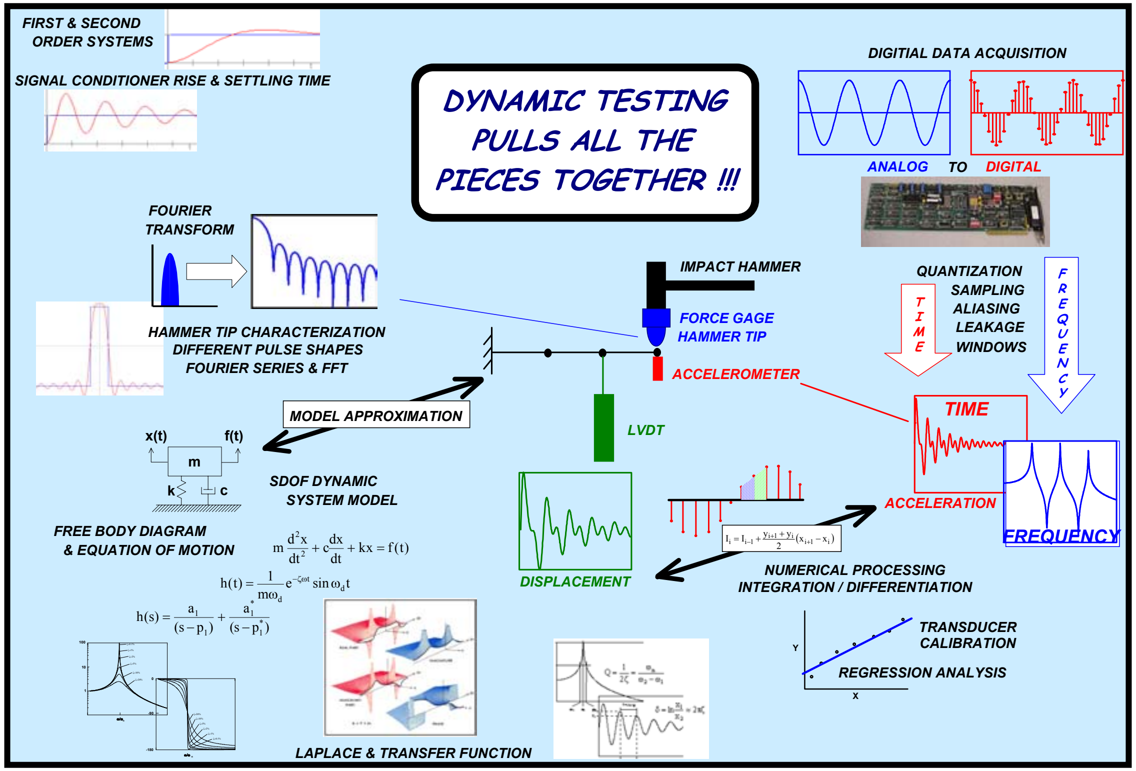
Tutorials cover a wide assortment of integrated material

Matlab GUIs are available for download

Acknowledgements

This project is partially supported by
NSF Engineering Education Division Grant EEC-0314875

Multi-Semester Interwoven Project for Teaching Basic Core STEM Material Critical for Solving Dynamic Systems Problems



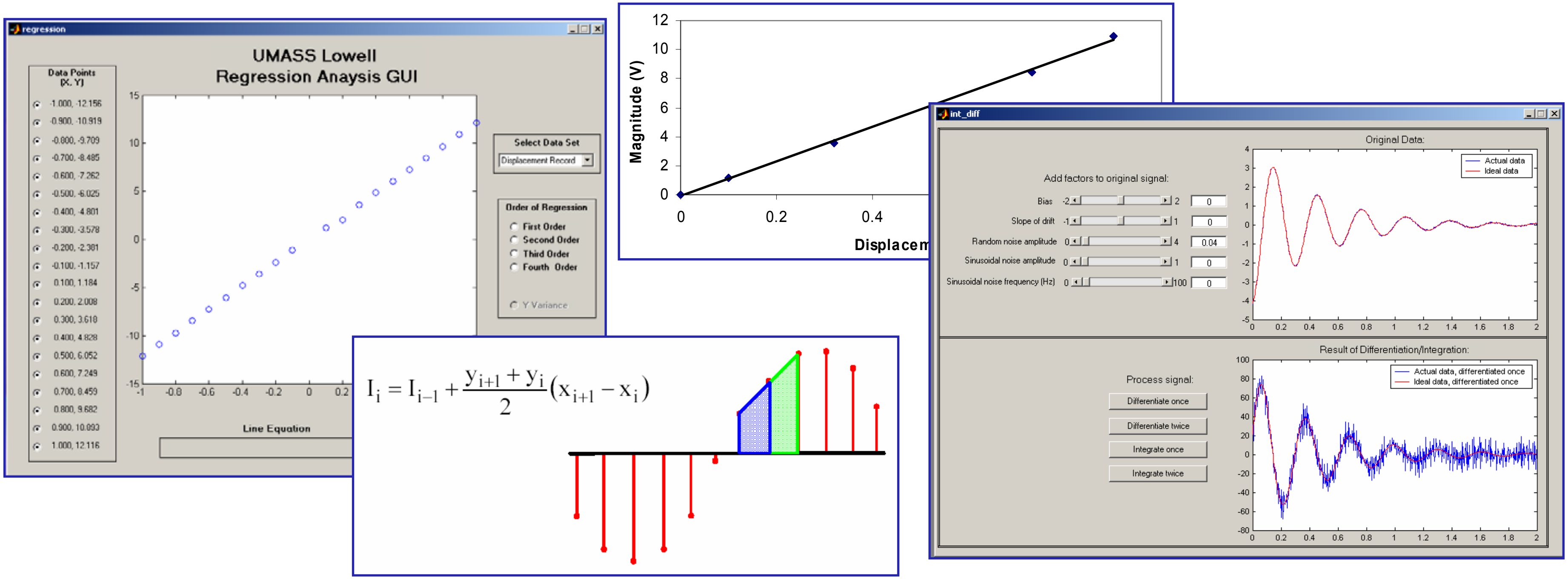
$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=f(t)$$

$$h(t)=\frac{1}{m\omega_d}e^{-\zeta\omega t}\sin\omega_d t$$

$$h(s)=\frac{a_1}{(s-p_1)}+\frac{a_1^*}{(s-p_1^*)}$$

Dr. Peter Avitabile, Dr. John McKelliget

Interweaving Numerical Methods Techniques in Multi-Semester Projects



$$I_i = I_{i-1} + \frac{y_{i+1} + y_i}{2}(x_{i+1} - x_i)$$

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Mechanical Engineering Department
University of Massachusetts Lowell