Phys 2690 Solution 9 Honors Physics III

(1) $$n(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

Substitute $A = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2}$

and $B = \frac{m}{2kT}$

$\therefore \; n(v) = A v^2 e^{-Bv^2}$

Check to see if $\int_0^\infty n(v)\,dv = N$

$$\int_0^\infty n(v)\,dv = A\int_0^\infty v^2 e^{-Bv^2}\,dv$$

$$= A \cdot \frac{\sqrt{\pi}}{4B^{3/2}}$$

$$= 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} \cdot \frac{\sqrt{\pi}}{4} \left(\frac{2kT}{m}\right)^{3/2}$$

$$= N$$

(a) $$v_{avg} = \frac{\int_0^\infty v\, n(v)\,dv}{\underbrace{\int_0^\infty n(v)\,dv}_{= N}}$$

$$= \frac{A\int_0^\infty v^3 e^{-Bv^2}\,dv}{N}$$

$$= \frac{A}{N}\cdot\frac{1}{2B^2} = \frac{4\pi N}{N}\left(\frac{m}{2\pi kT}\right)^{3/2} \cdot \frac{4k^2T^2}{2m^2}$$

$$\boxed{= \sqrt{\frac{8kT}{\pi m}}}$$

(b) $\boxed{v_{rms} = \sqrt{\overline{v^2}}}$ $\quad \overline{v^2} = \int_0^\infty v^2 n(v)\,dv \Big/ N$

$$= \frac{A}{N}\int_0^\infty v^4 e^{-Bv^2}\,dv$$

$$= \frac{A}{N}\cdot\frac{3\sqrt{\pi}}{8B^{5/2}}$$

$$= \frac{4\pi N}{N}\cdot\left(\frac{m}{2\pi kT}\right)^{3/2}\cdot\frac{3\sqrt{\pi}}{8}\left(\frac{2kT}{m}\right)^{5/2}$$

$$= \frac{3kT}{m}$$

$$\therefore \boxed{v_{rms} = \sqrt{\frac{3kT}{m}}}$$

(c) Most probable speed is when $\frac{dn(v)}{dv} = 0$

$$\frac{d}{dv}\left(Av^2 e^{-Bv^2}\right) = 0$$

$$2Av e^{-Bv^2} - ABv^2 e^{-Bv^2}(2v) = 0$$

$$\text{or} \quad 2Av e^{-Bv^2} = 2ABv^3 e^{-Bv^2}$$

$$\text{or} \quad v^2 = 1/B = \frac{2kT}{m}$$

$$\boxed{v_p = \sqrt{\frac{2kT}{m}}}$$

(d) We can also write the formulae as $\sqrt{\frac{2RT}{M}}$ etc. where M is the molar mass of oxygen molecules.

$\therefore \quad v_{avg} = \sqrt{\frac{8}{\pi}} \sqrt{\frac{RT}{M}} \qquad M = 32\text{ g or } 0.032\text{ kg}$

$$= 1.60 \sqrt{\frac{RT}{m}} = 1.595 \sqrt{\frac{8.31 \times 300}{0.032}} \text{ m/s}$$

$$= 1.595 \times 279.1 \text{ m/s}$$

$$= \boxed{445 \text{ m/s}}$$

$$v_{rms} = \sqrt{3} \cdot \sqrt{\frac{RT}{M}} = 1.732 \times 279.1 \text{ m/s}$$

$$= \boxed{483 \text{ m/s}}$$

$$v_p = \sqrt{2} \sqrt{\frac{RT}{M}} = 1.414 \times 279.1 \text{ m/s}$$

$$= \boxed{395 \text{ m/s}}$$

(2) $pV = Nk_BT \Rightarrow \frac{N}{V} = \frac{p}{k_BT}$, $p = 1\text{ atm} = 101.3 \times 10^3$ Pa

$v_{rms} = \sqrt{\frac{3k_BT}{m}}$, $\lambda = \frac{1}{\sqrt{2}\,\pi\,(N/V)\,d^2}$

$d = 314\text{ pm} = 3.14 \times 10^{-10}$ m

Collision rate $= \frac{v_{rms}}{\lambda} = 6.0 \times 10^9 \text{ s}^{-1}$

$$m_{nitrogen} = \frac{0.028 \text{ kg}}{6.02 \times 10^{23}}$$

$$\text{Rate} = \sqrt{\frac{3k_BT}{m}} \cdot \sqrt{2}\,\pi \left(\frac{N}{V}\right) d^2 = \sqrt{\frac{6}{mk_BT}}\,\pi p d^2$$

$$T = \frac{6\pi^2 p^2 d^4}{m\,k_B\,(\text{Rate})^2}$$

$$= \frac{6\pi^2 \times (101.3 \times 10^3)^2 \times (3.14 \times 10^{-10})^4 \times 6.02 \times 10^{23}}{.028 \times 1.38 \times 10^{-23} \times (6 \times 10^9)^2}$$

$$= \boxed{256 \text{ K}}$$

(3) (a) Prob. of finding all N balls in box 1 $= \left(\frac{1}{2}\right)^N$

[20–36] " " " " " " " " 2 $= \left(\frac{1}{2}\right)^N$

So " " " " " " " either box 1 or box 2 $= 2\left(\frac{1}{2}\right)^N$

$$2\left(\frac{1}{2}\right)^N = \frac{80}{10{,}000} = 0.008$$

$$(0.5)^N = 0.004$$

$$N\log(0.5) = \log(0.004)$$

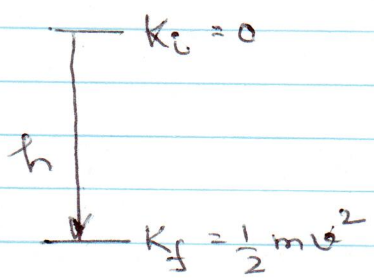
$$N = \frac{\log(0.004)}{\log(0.5)} = 7.97$$

$$\therefore \boxed{N = 8}$$

(b) In equilibrium, $N_1 = N_2 = \frac{N}{2} = \boxed{4}$ in each box

(4)

[20–38]



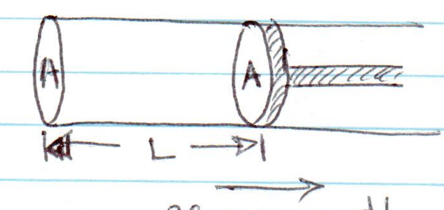
$$\overset{\Delta K}{\frac{1}{2}mv^2} = \overset{\Delta U}{mgh} = \epsilon = \frac{5}{2}k_B T$$

$$h = \frac{5k_BT}{2mg} = \frac{5\times1.38\times10^{-23}\times300\times6.02\times10^{23}}{2\times0.032\times9.81}$$

$$\left[m_{O_2} = \frac{0.032\,\text{kg}}{6.02\times10^{23}}\right] \qquad h = \boxed{1.99\times10^4\,\text{m}} \approx \boxed{20\text{ km}}$$

(5)

[20–44]



$v_{piston} = \frac{dL}{dt}$

Isobaric expansion

p = constant

$v_{rms} = 450$ m/s

$v_{piston} = 0.50$ m/s

$L = 1.5$ m

$$v_{rms} = \sqrt{\frac{3p}{(N/V)m}} = \sqrt{\frac{3pV}{M}} \quad \text{where } M = Nm \text{ and } V = AL$$

$$\frac{dv_{rms}}{dt} = \frac{d}{dt}\sqrt{\frac{3pAL}{M}} = \frac{d}{dt}\sqrt{\frac{3pA}{M}}\cdot L^{1/2}$$

$$\therefore \frac{dv_{rms}}{dt} = \frac{1}{2}L^{-1/2}\frac{dL}{dt}\sqrt{\frac{3pA}{M}} = \frac{1}{2L}\sqrt{\frac{3pAL}{M}}\cdot\frac{dL}{dt}$$

$$= (v_{rms}\,v_{piston})/2L$$

$$dv_{rms}/dt = (450\times0.50)/1.5 = \boxed{75\text{ m/s}^2}$$

(6) [20-52]

Number of moles $n_{Helium} = \dfrac{2.0\,g}{4.0\,g/mol} = 0.5$ mol

Number of moles $n_{oxygen} = \dfrac{8.0\,g}{32\,g/mol} = 0.25$ mol

Helium is monatomic and oxygen is diatomic

(a)

Initial thermal energies

$$E_{th,i,Helium} = n_{Helium}\left(\frac{3}{2}RT\right) = \left(0.5 \times \frac{3}{2} \times 8.31 \times 300\right)\text{J} = \boxed{1870\ \text{J}}$$

$$E_{th,i,Oxygen} = n_{oxygen}\left(\frac{5}{2}RT\right) = \left(0.25 \times \frac{5}{2} \times 8.31 \times 600\right)\text{J} = \boxed{3116\ \text{J}}$$

(b) On mixing, $E_{total} = 1870\,\text{J} + 3116\,\text{J} = 4986\,\text{J}$

Common temperature T_f

$$n_{Helium}\left(\frac{3}{2}RT_f\right) + n_{oxygen}\left(\frac{5}{2}RT_f\right) = 4986\ \text{J}$$

$$T_f = \frac{4986\ \text{J}}{\frac{1}{2}R(3n_{Helium} + 5n_{oxygen})} = \frac{4986\ \text{J}}{\frac{1}{2} \times 8.31 \times (3\times0.5 + 5\times0.25)}$$

$$436.4\,\text{K} = \boxed{436\ \text{K}}$$

(c) So $E_{th,f,Helium} = \left(0.5 \times \frac{3}{2} \times 8.31 \times 436.4\right)\text{J} = \boxed{2720\ \text{J}}$

$$E_{th,f,oxygen} = \left(0.25 \times \frac{5}{2} \times 8.31 \times 436.4\right)\text{J} = \boxed{2267\ \text{J}}$$

(c) Heat energy transferred $= (3116 - 2267)\text{J} = 849\,\text{J}$
(from oxygen to helium)

$\approx$

or $(2720 - 1870)\text{J} = \boxed{850\ \text{J}}$

(7) [20-58] Monatomic gas adiabatically compressed to $\left(\frac{1}{8}\right)$ of its initial volume.

(a) $v_{rms} \propto \sqrt{T}$

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \Rightarrow T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = T_i (8)^{5/3-1} = 4T_i$$

$\therefore$ v_{rms} goes up by a factor of 2

(b) $\lambda \propto \frac{1}{(N/V)} \propto V$

So mean free path goes down by a factor of 8

(c) $E_{th} \propto T$

So E_{th} goes up by a factor of 4.

(d) $C_V = \frac{3}{2}R$ remains a constant.

(8) [20-62] $\frac{1}{2} m v_{esc}^2 = \frac{G M_E m}{R_E} \Rightarrow v_{esc} = 11.2$ km/s

$$v_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

Temp T at which $v_{rms} = v_{esc} = v \Rightarrow$

$$T = \frac{m v^2}{3 k_B}$$

$m_{nitrogen} = \frac{0.028 \text{ kg}}{6.02 \times 10^{23}}$ $\quad m_{hydrogen} = \frac{0.002}{6.02 \times 10^{23}}$

$$T_{nitrogen} = \frac{0.028 \times \left[(11.2 \times 10^3) \text{ m/s}\right]^2}{6.02 \times 10^{23} \times 3 \times 1.38 \times 10^{-23}} = \boxed{1.41 \times 10^5 \text{ K}}$$

$$T_{hydrogen} = T_{nitrogen} \times \frac{2}{28} = \boxed{1.01 \times 10^4 \text{ K}}$$

(c) $\langle \epsilon \rangle = \frac{3}{2} k_B T = \frac{3}{2} \times 1.38 \times 10^{-23} \times 273$ J

average translational kinetic energy $= 5.7 \times 10^{-21}$

For nitrogen, KE for escape $= \frac{1}{2} m_{nitrogen} \times v_{esc}^2$

$$= \frac{1}{2} \times \frac{0.028}{6.02 \times 10^{23}} \times (11.2 \times 10^3)^2$$

$$= 2.9 \times 10^{-18} \text{ J}$$

For hydrogen, KE for escape $= 2.9 \times 10^{-18}$ J $\times \frac{m_{hydrogen}}{m_{nitrogen}}$

$$= 2.9 \times 10^{-18} \times \frac{2}{28} \text{ J}$$

$$= 2.1 \times 10^{-19} \text{ J}$$

$\frac{\langle \epsilon \rangle}{K_{esc}} = \frac{5.7 \times 10^{-21}}{2.9 \times 10^{-18}} = 0.2\%$ for nitrogen

and $\frac{5.7 \times 10^{-21}}{2.1 \times 10^{-19}} = 2.7\%$ for hydrogen

Since this ratio needs to be less than 1% for the gas not to escape, nitrogen stays and hydrogen leaves the earth's atmosphere.

(9) (a) Air   80% N_2,   20% O_2 (both diatomic)

[20-64]

$$E_{th} = \frac{5}{2} N_{total} K_B T$$

Vol of room $= (2 \times 2 \times 2)\ m^3$

$$pV = N_{total} k_B T$$

$$\therefore E_{th} = \frac{5}{2} pV = \frac{5}{2} \times 101.3 \times 10^3 \times 2 \times 2 \times 2 \quad J$$

$$= \boxed{2.03 \times 10^6\ J}$$

(b) $E_{ball} = mgh = 1\ kg \times 9.8\ m/s^2 \times 1\ m = 9.8\ J$

Fraction of thermal room energy needed $= \frac{9.8}{2.03 \times 10^6} = \boxed{4.8 \times 10^{-6}}$

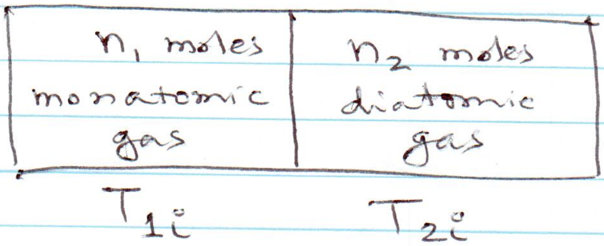
(c) $$\frac{\Delta E_{th}}{E_{th}} = \frac{\Delta T}{T}$$

$$\therefore \Delta T = \frac{\Delta E_{th}}{E_{th}} \times T = -4.8 \times 10^{-6} \times 273$$

$$= -.0013\ K$$

(d) Highly improbable event, lower entropy. violates second law of thermodynamics.

(10) (a)
[20-66]



$$E_{tot} = E_{1i} + E_{2i} = \frac{3n_1RT_{1i} + 5n_2RT_{2i}}{2}$$

$$E_{final} = E_{tot} = \frac{3n_1RT_f + 5n_2RT_f}{2}$$

$$\therefore \quad (3n_1 + 5n_2)T_f = 3n_1T_{1i} + 5n_2T_{2i}$$

(b) $$T_f = \frac{3n_1T_{1i} + 5n_2T_{2i}}{3n_1 + 5n_2}$$

$$= \frac{2(E_{1i} + E_{2i})}{R(3n_1 + 5n_2)}$$

(a) $$\therefore \quad E_{1f} = \frac{3n_1RT_f}{2} = \frac{3n_1}{3n_1+5n_2}(E_{1i} + E_{2i})$$

$$E_{2f} = \frac{5}{2}n_2RT_f = \frac{5n_2}{3n_1+5n_2}(E_{1i} + E_{2i})$$

(c) $n_1 = \frac{2.0g}{4.0g/mol}$ of He $= 0.5$ mol at 300 K

$n_2 = \frac{8.0g}{32.0g/mol}$ of O_2 $= 0.25$ mol at 600 K

$$T_f = \frac{(3\times0.5\times300)+(5\times0.25\times600)}{(3\times0.5)+(5\times0.25)} = \boxed{436\ K}$$

for He
$$Q_1 = n_1C_V\Delta T = (0.5\,mol)\times(12.5\,J/mol\,K)\times(436-300)K \approx \boxed{850\,J}$$

for O_2
$$Q_2 = n_2C_V\Delta T = 0.25\times20.5\times(436-600) \boxed{\approx -850\,J}$$