Quadratic Functions

$$f(x) = ax^2 + bx + c$$

Graphs of quadratics functions are parabolas opening up if $a > 0$, and down if $a < 0$.

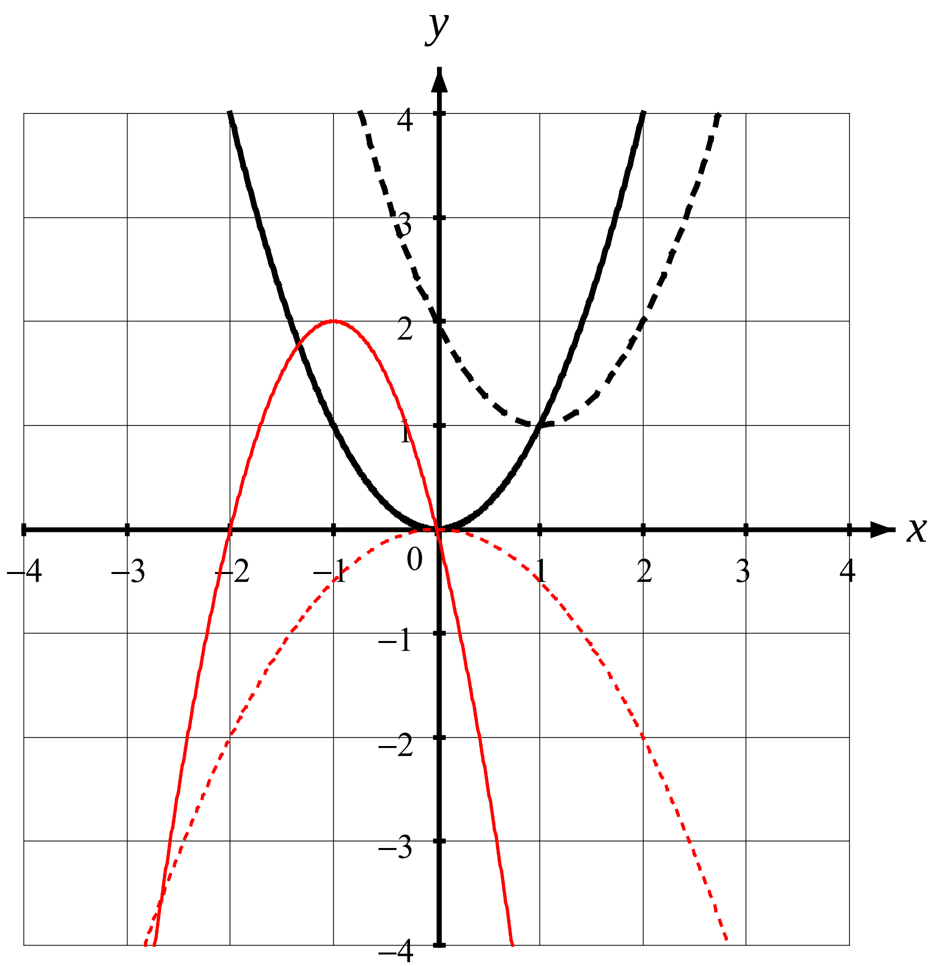
Examples:

$y = x^2$

$y = x^2 - 2x + 2$

$y = -2x^2 - 4x$

$y = -\dfrac{x^2}{2}$



Notes:

1) Quadratic functions always have an axis of symmetry, that is a vertical line about which the graph is symmetric.

   This line is $x = -\frac{b}{2a}$

2) They also have a point where they "turn around". This point is called the vertex and it occurs where $x = -\frac{b}{2a}$, and $y = c - \frac{b^2}{4a}$.

3) They definitely intersect the y-axis at $(0,c)$.

   They may or may not intersect the x –axis. This depends upon the discriminant, $b^2 - 4ac$.

How to graph a quadratic function $f(x) = ax^2 + bx + c$

1) Check the sign of a. If $a > 0$, the graph is curved up, for $a < 0$ it's curved down.
2) Determine the line of symmetry by computing $x = -\frac{b}{2a}$
3) Determine the vertex using $x = -\frac{b}{2a}$.
4) Find the y-intercept. (Set $x = 0$, and so $y = c$.)
5) Determine x-intercepts by solving $ax^2 + bx + c = 0$.

Example: Graph $f(x) = x^2 - 4x + 3$.

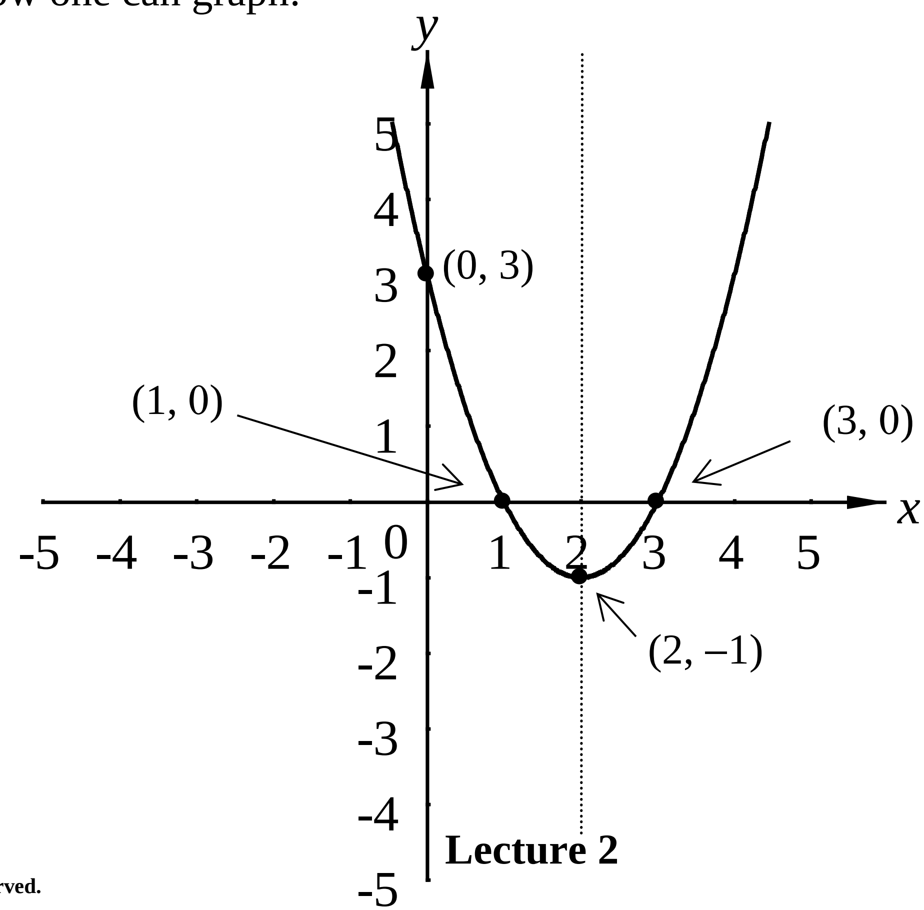
Here $a = 1$, $b = -4$, and $c = 3$, so $-\dfrac{b}{2a} = -\dfrac{-4}{2} = 2$. Since $a > 0$, the graph is a parabola opening upwards. The axis of symmetry is $x = 2$.

The vertex is when $x = 2$, in which case $y = -1$. So, it is the point $(2, -1)$.

The y-intercept is $(0, 3)$.

The x-intercepts occur where $x^2 - 4x + 3 = 0$, or $x = 1$, and $x = 3$.

Now one can graph:



Example: Graph $f(x) = -2x^2 - 4x + 6$.

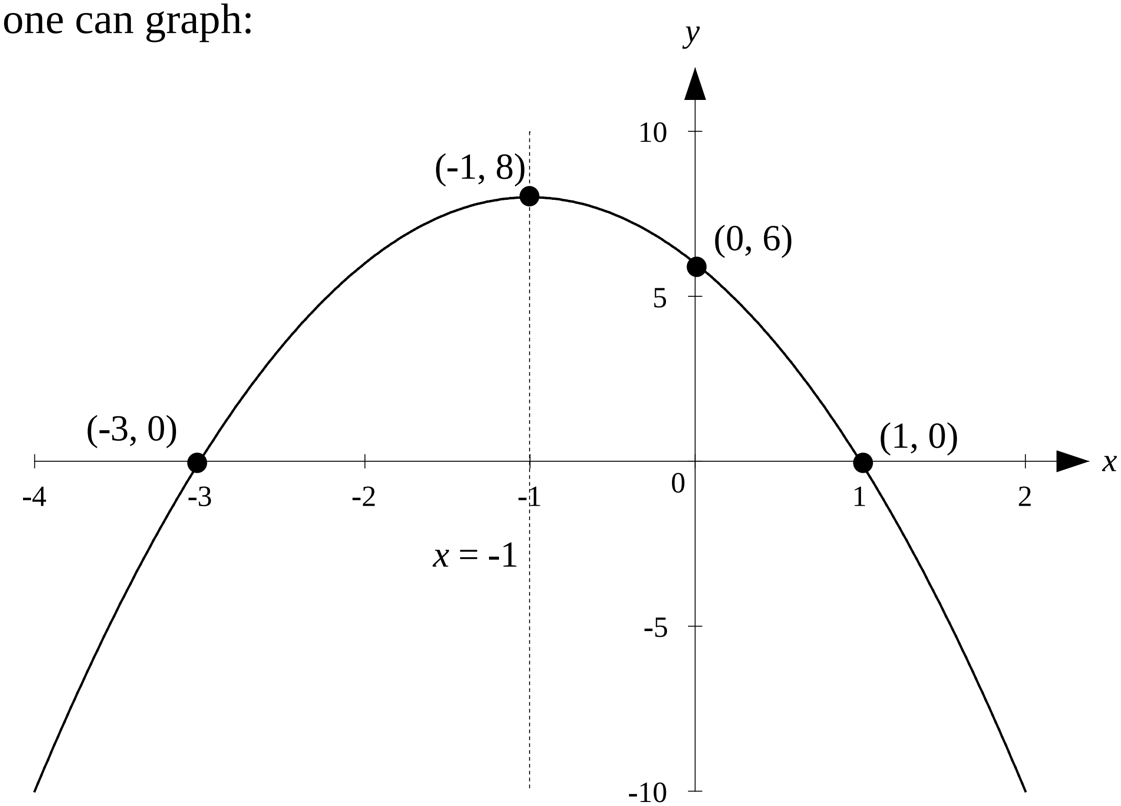
Here $a = -2$, $b = -4$, and $c = 6$, so $-\frac{b}{2a} = -\frac{(-4)}{2(-2)} = -1$. Since $a < 0$, the graph is a parabola opening downwards.

The vertex is when $x = -1$, in which case $y = 8$. So, it is the point $(-1, 8)$.

The y-intercept is (0, 6).

The x-intercepts occur where $-2x^2 - 4x + 6 = 0$, or $x = 1$, and $x = -3$.

Now one can graph:

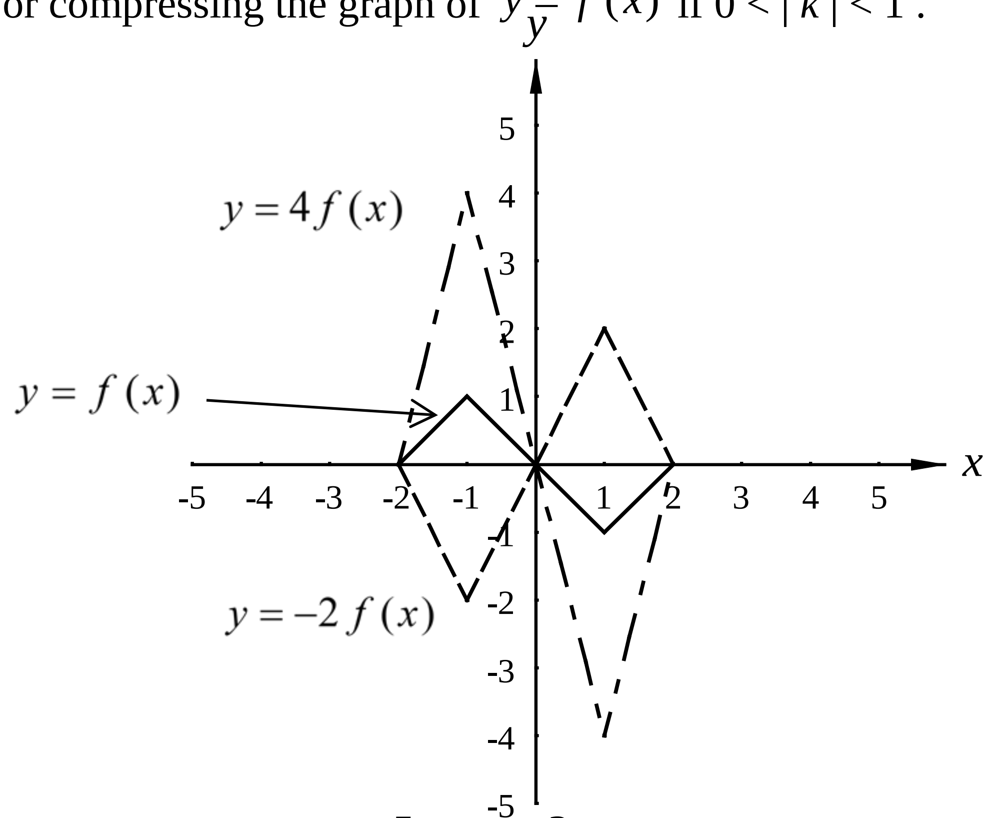


Scaling Functions

Vertical Scaling: Suppose $k > 0$, then one may obtain the graph of $y = k\ f\,(x)$ by stretching the graph vertically of $y = f\,(x)$ if $k > 1$, or compressing the graph (vertically) of $y = f\,(x)$ if $0 < k < 1$.

If $k < 0$, then one may obtain the graph of $y = k\ f\,(x)$ by first rotating the graph about the x-axis, then by stretching the graph of $y = f\,(x)$ vertically if $|\,k\,| > 1$, or compressing the graph of $y = f\,(x)$ if $0 < |\,k\,| < 1$.

Examples:



$$y = x^2$$

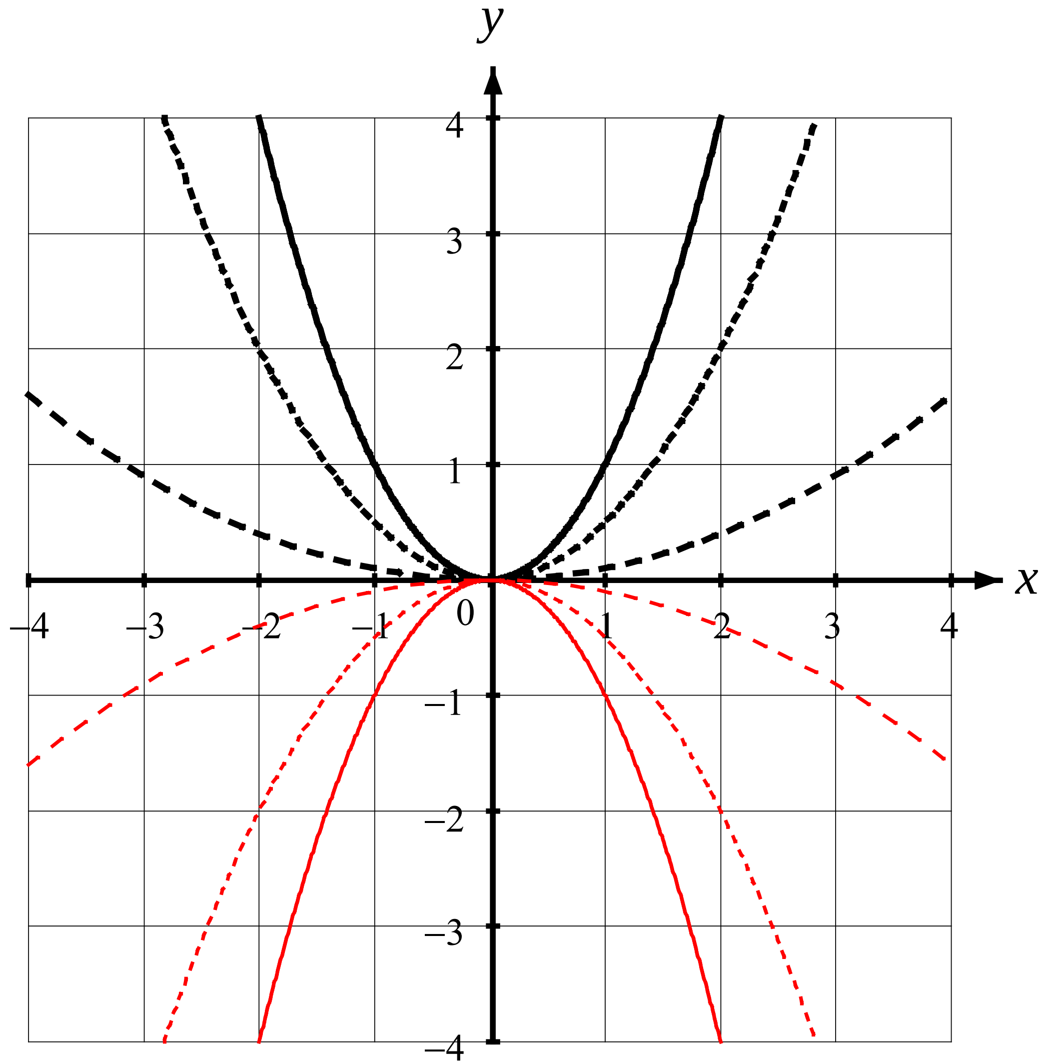
$$y = \frac{1}{2}x^2$$

$$y = \frac{1}{10}x^2$$

$$y = -x^2$$

$$y = -\frac{1}{2}x^2$$

$$y = -\frac{1}{10}x^2$$



Horizontal Scaling:

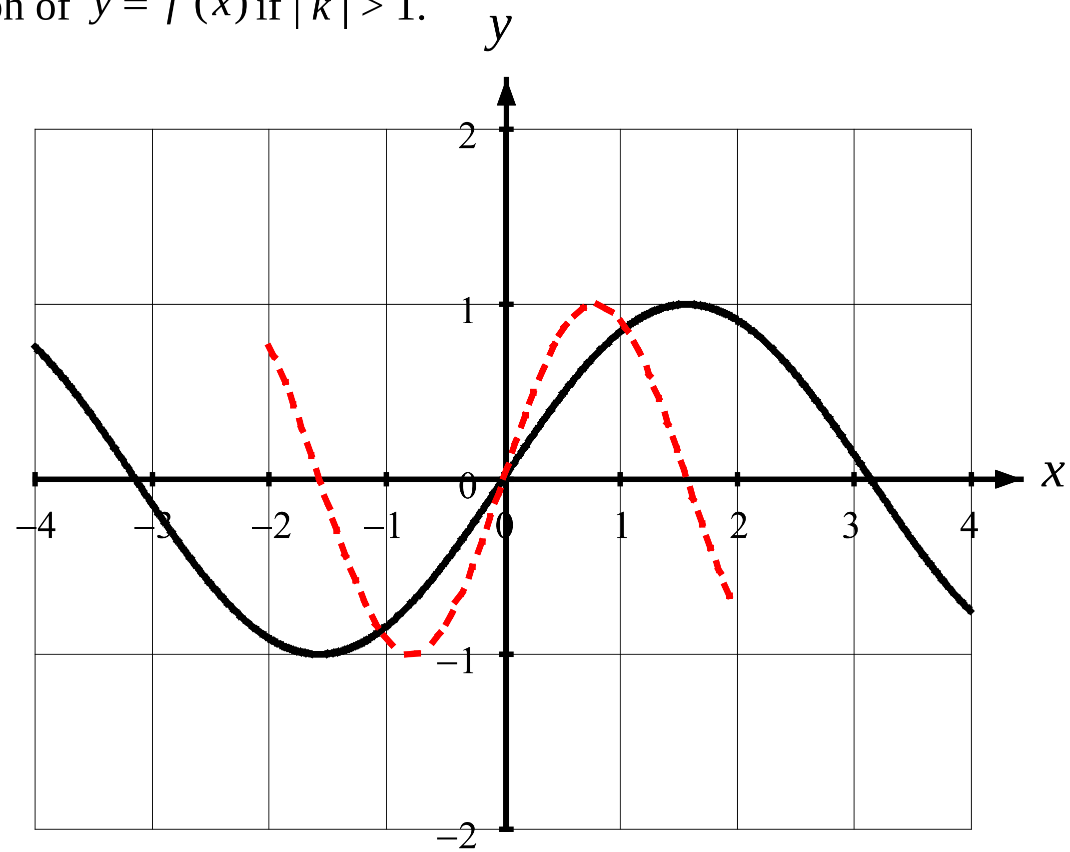
Suppose $k > 0$, then one may obtain the graph of $y = f(kx)$ by stretching the graph of $y = f(x)$ horizontally if $0 < k < 1$, or compressing the graph of $y = f(x)$ if $k > 1$.

If $k < 0$, then one may obtain the graph of $y = f(kx)$ by first rotating the graph about the y-axis, then by stretching the graph of $y = f(x)$ horizontally if $0 < |k| < 1$, or by compressing the graph of $y = f(x)$ if $|k| > 1$.

Example:

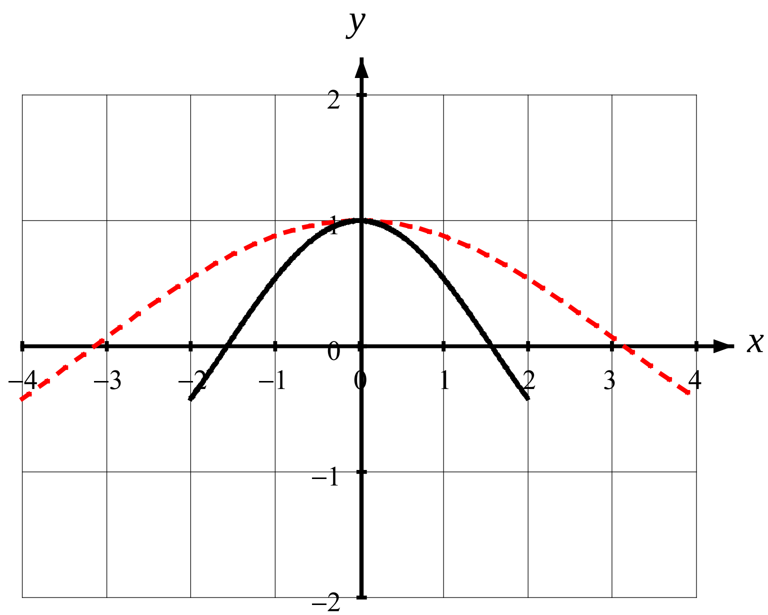
$y = f(x)$

$y = f(2x)$



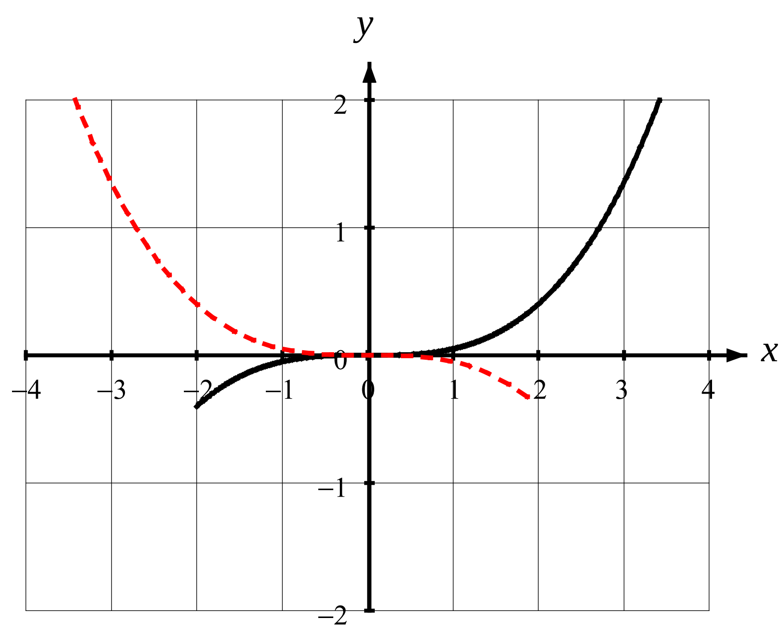
$y = f(x)$

$y = f(x/2)$



$y = f(x)$

$y = f(-x)$



<u>Shifting Functions</u>

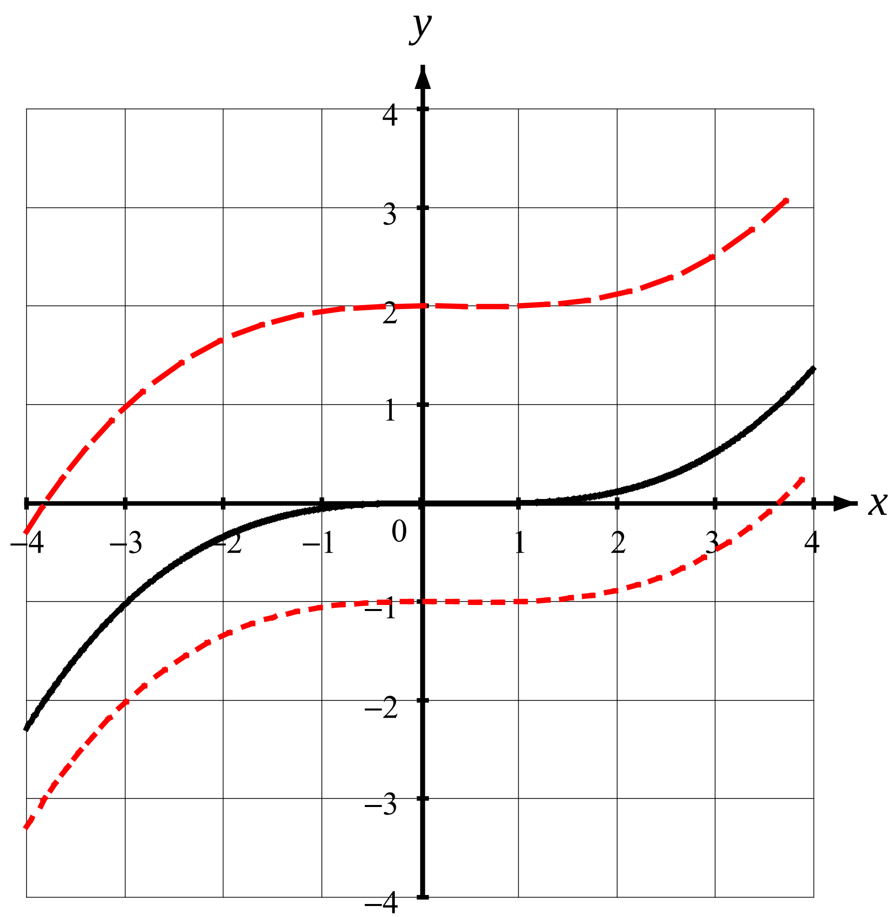
<u>Vertical Shifts:</u> Suppose $a > 0$, then one may obtain the graph of $y = f(x) + a$ by shifting the graph of $y = f(x)$ up a units.
The graph of $y = f(x) - a$ is obtained from the graph of $y = f(x)$ by shifting it down a units.

Examples:

$y = f(x) + 2$

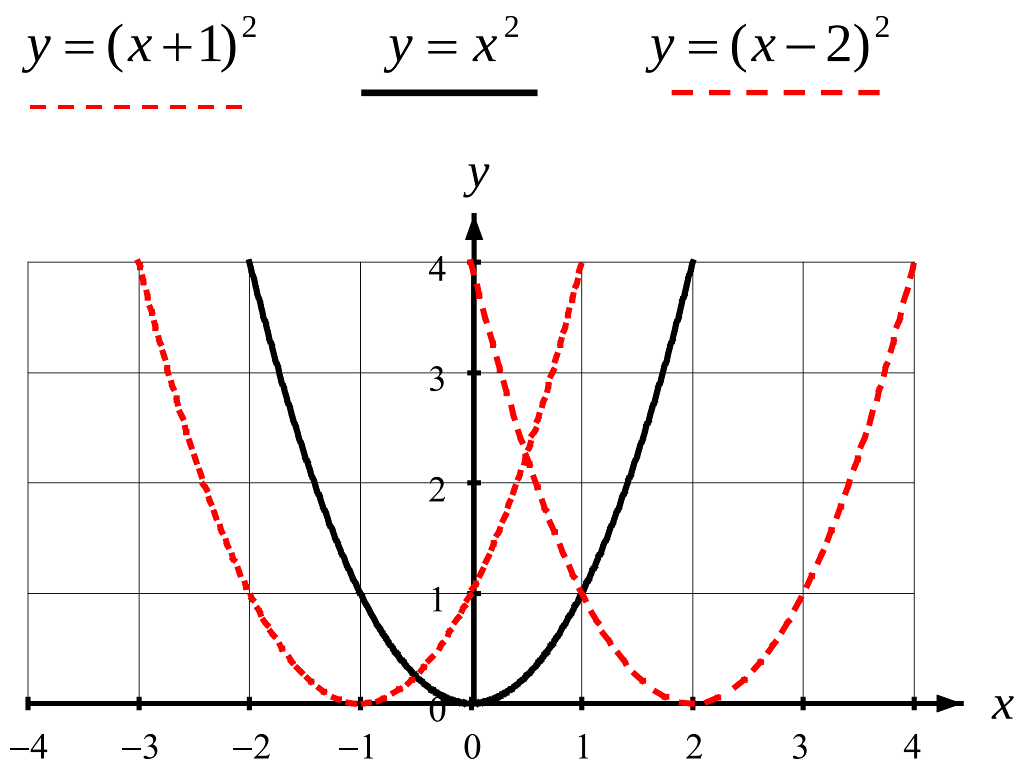
$y = f(x)$

$y = f(x) - 1$



Horizontal Shifts: Suppose $a > 0$, then one may obtain the graph of $y = f(x - a)$ by shifting the graph of $y = f(x)$ right a units.
The graph of $y = f(x + a)$ is obtained from the graph of $y = f(x)$ by shifting it left a units.

Example:

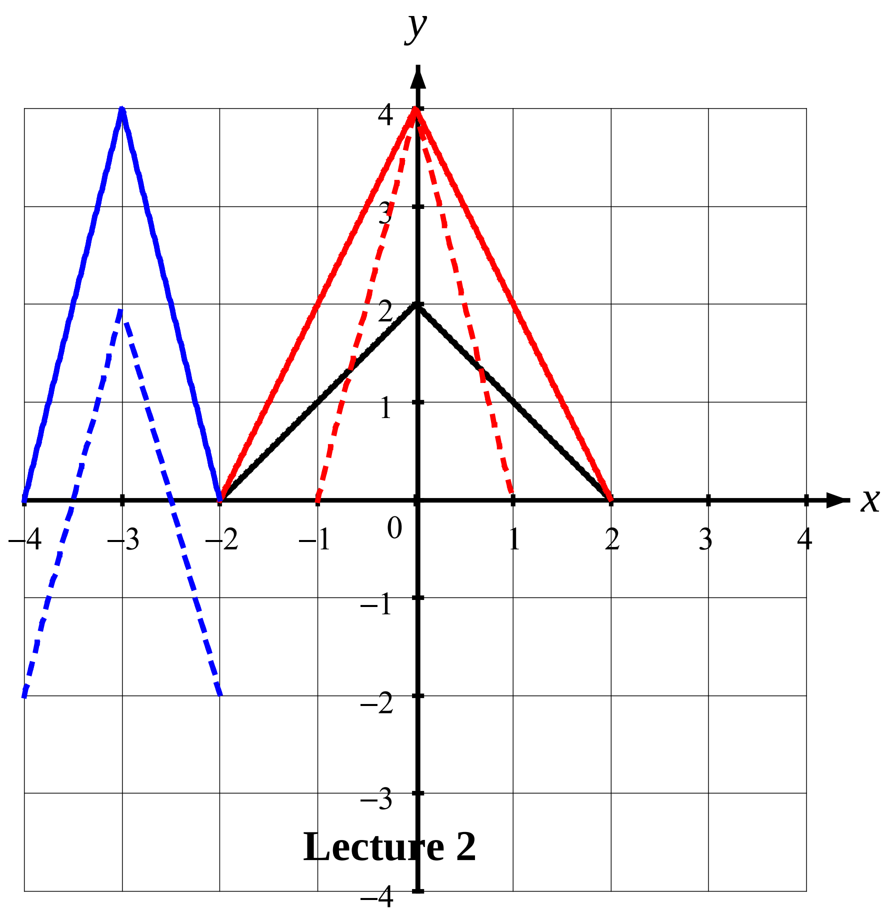


Putting It All Together:

Example: Given the function $f(x)$, shown in black below, graph the function $g(x) = 2f(2(x+3)) - 2$.

(red solid line)	1)	First graph $2f(x)$	Vertical stretch of 2.
(red dotted line)	2)	Then graph $2f(2x)$	Horizontal compression by 2.
(blue solid line)	3)	Then graph $2f(2(x+3))$	Horizontal shift left 3 units.
(blue dotted line)	4)	Finally graph $g(x) = 2f(2(x+3)) - 2$	Vertical shift down 2 unit.

$f(x)$



Even and Odd Functions:

A function *f* is **even** if $f(-x) = f(x)$ for all *x* in its domain.

A function *f* is **odd** if $f(-x) = -f(x)$ for all *x* in its domain.

Examples: $f(x) = x^4$ is even since $f(-x) = (-x)^4 = x^4 = f(x)$.

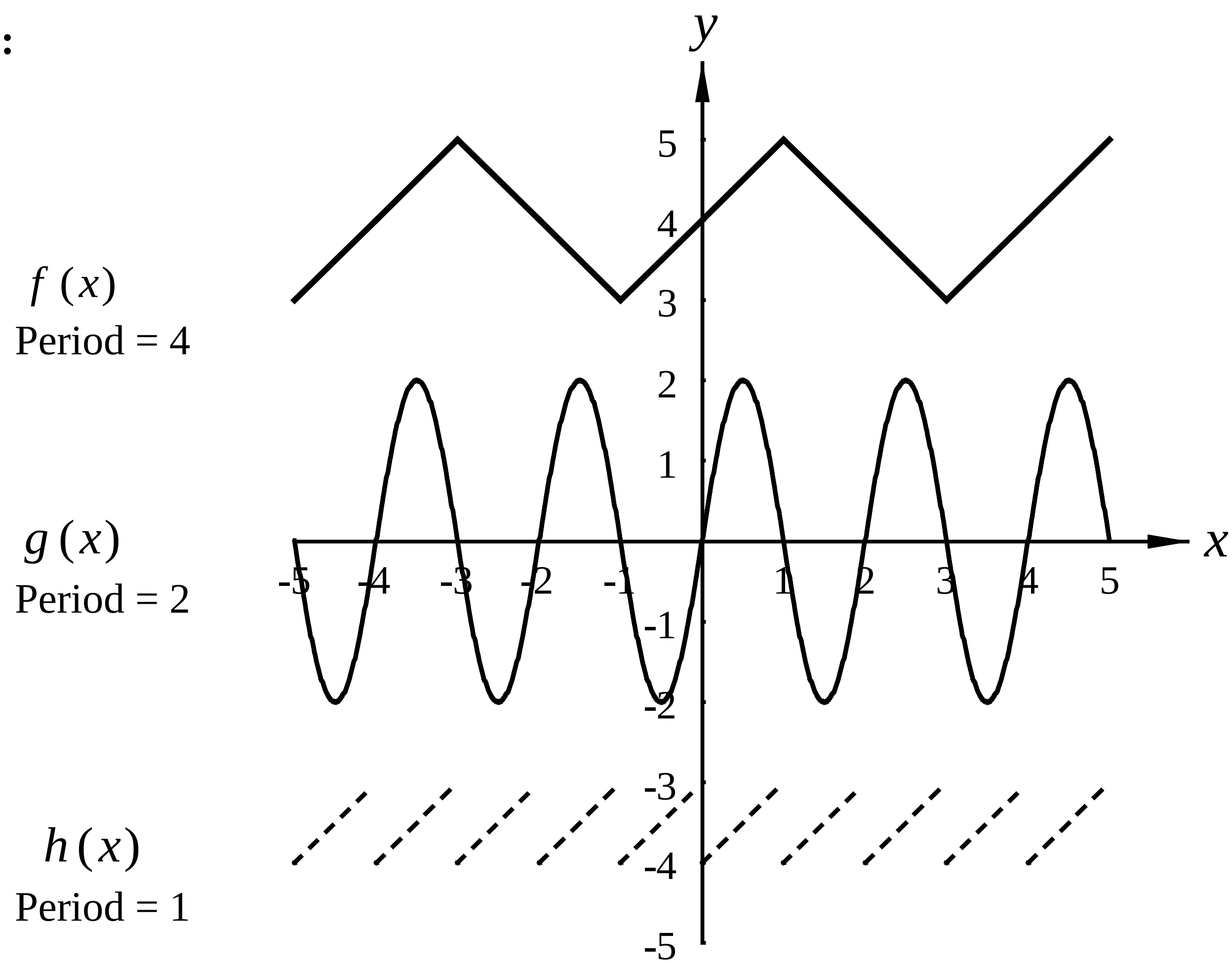
$f(x) = x^3$ is odd since $f(-x) = (-x)^3 = -x^3 = -f(x)$.

Notes:
1) Even functions are symmetric about the *y*-axis.
2) Odd functions are symmetric through the origin.
3) An even function times an even function is also even.
4) An even function times an odd function is **ODD!**
5) An odd function times an odd function is **EVEN!**
6) A function can be neither even nor odd.
   Take for example $f(x) = x^4 + x^3$

Periodic Functions:

A function is periodic with period P if $f(x+P) = f(x)$ holds for all x in the domain of f. The smallest such number P is called the **period** of f.

Examples:



Final Function and Graph Summary:

Vertical Line Test:

Since a function must have one and only one value associated with each element in its domain, if a any vertical line crosses a graph more than once, the graph cannot be that of a function.

Intercepts:

Any place that the graph of a function intersects the y-axis is called the y-intercept. If the point $(0, b)$ is on the graph of the function one says either $(0, b)$ is the y-intercept, or $y = b$ is the y-intercept. This can happen at most once.

Any place that the graph of a function intersects the x-axis is called the x-intercept. If the point $(a, 0)$ is on the graph of the function one says either $(a, 0)$ is the x-intercept, or $x = a$ is the x-intercept. This can occur any number of times.

Roots

Any time the graph of the function $y = f(x)$ has an x-intercept represents a place where $f(x) = 0$. This place is called a root of the function. This is useful in graphing functions.