Lecture Notes 1 Part A: Call and Put Options

Introduction to Financial Derivatives

This introductory lecture is mostly about call and put options. Since everyone understands that calls and puts are options, they are often just referred to as "calls" and "puts". Options are just one kind of financial derivative, so I will start by discussing derivatives in general.

Let's look at just a few of these definitions before I define the objectives of this learning module:

Derivative An asset whose performance depends on another asset; examples include options, forwards, futures, and swaps.

Option A contract between buyer and seller of an option, which gives the buyer the right but not the obligation to buy or sell an asset at a later date, for a price agreed upon at the time the contract is struck. The right to buy the asset is a **call option**, while the right to sell the asset is a **put option**.

As you can see from the above definition there are two points in time at which "something" may change hands. Consider a call on a share of stock: First, the option is purchased, and then at some future point in time, there is a possibility that the share of stock also changes hands.

Options are derivatives because their value is, in part, derived from the value of another asset, in this case the value of the share of stock. Options and other derivatives are created by a contract between a buyer and seller, so options are often referred to as an **option contract**.

Let's continue with some more definitions, starting with a more exact description of calls and puts:

Call Option Contract between the buyer and seller of the call, giving the call buyer the right, but not the obligation, to buy some specified asset from the call seller at a specified future date but at a price agreed upon today!

The specified asset can be anything, but we will often use a share of stock as an example.

Put Option Similar to above, but giving the put buyer the right to sell the agreed upon asset to the put seller.

How about other kinds of derivatives?

Forward A contract between buyer and seller which gives the buyer the right and the obligation to buy an asset at a later point in time at a price agreed upon today.

The difference between options and forwards is the "obligation" to carry out the contract. With a forward contract, the buyer must buy and the seller must sell at the agreed upon future date.

Futures A futures contract is just an organized market's version of a forward contract where procedures are in place to reduce the risk of default. By organized market we mean an exchange like the Chicago Board of Trade.

Swap A contract in which each side gives up an asset or a set of cash flows to the other. A swap can sometimes be compared to a pair of linked forward contracts.

Definitions for Calls and Puts

We now turn to some definitions specific definitions related to options. Unfortunately, sometimes there exists two definitions for the same concept, and you will have to learn both of them since you are bound to run into both.

Call Option A contract conveying the right to buy an asset on or before a stated future date for a stated price.

Put Options A contract conveying the right to sell an asset on or before a stated future date for a stated price.

Underlying Asset The asset stated in the contract. In the examples below, the underlying asset will be a share of stock.

Strike Price or Exercise Price The stated price in the contract.

Exercising Taking advantage of the right conveyed by the option and buying (for the case of a call) or selling (for the case of the put) the underlying asset in exchange for the exercise price.

Expiration Date or Maturity Date The last day on which the option may be exercised.

American Option Can be exercised on or before the expiration date.

European Option Can only be exercised on the expiration date.

Option Holder The person buying the contract.

Option Writer The person selling the contract.

Option Premium The market price of the option.

Everything else being the same, which option should be worth more, the American or the European option?

Study Hint: When you run into a question in these notes (***both bold face and italics***), stop reading for a moment and try to answer the question. This will test your understanding of the material. If you are have some difficulty coming up with an answer, try backing up a bit.

It seems that an American option can do everything a European option can do and some more! An American option should be worth more, although we will see that under certain circumstances the two types, American and European, will be identical in value. An American option certainly can't be worth less than a European. Incidentally, the geographic region in which these options trade is incidental to the type that they are, which is determined by the contract or the specifications of the options exchange.

Unlike stock, options have a finite life. One objective for this course is to learn the costs and benefits of various strategies which use options. From just the above definition of options it is easy to figure out the **value of the option** at expiration and also the **profit** to both the buyer and the seller of the option. The knowledge of possible profit or loss over the life of the option will help in understanding the use of the strategy.

Learning Objectives of this Module

1. Introduce derivatives in general, as well as **calls** and **puts** in detail.

2. Introduce some notation for options which will allow us to discuss **values** and **profits** in an equation form.

3. Introduce **graphs** of both the values and profits of options at expiration.

4. Learn the systematic method to plot linear functions.

The first three learning objectives are covered in Part A of the lecture notes, while the fourth is contained in Part B. You might be surprised to actually see this last learning objective in a finance course. The importance of this skill for derivatives will become evident by the end of the second learning module and you will create such graphs throughout the course. I am quite certain you have already learned how to plot linear functions; this is actually a review of the topic.

Equations provide the advantage of making very concise statements. Graphs of equations result in a picture, which can make the concise statement more clear than words alone. To get started, we introduce some notation. Then we present equations for values and profits at expiration. Although we have some graphs, we will not dwell extensively on how to create these graphs; we will save that for both the homework and the second learning module. But this will require the skills that can be picked up in Part B of the lecture, which presents graphing of linear functions without much discussion of calls and puts.

Notation

S_t Stock price at time t

C_t Call **value** at time t

P_t Put **value** at time t

C_t^{MP}, P_t^{MP} Call and Put **market prices (*MP*)** at time t

X Exercise Price

T Expiration Date

Where we are at a given point in time is very important for most values in finance. The small subscript t used above indicates the point in time. For the most part, we will be interested in two particular points in time: The point when we initially purchase or sell a security, which we will denote as t = 0, and the expiration of an option contract, which we will denote as t = T. We can indicate by a time line diagram the passage of time:

Time Line:

$t = 0$	$t = T$
S_0	S_T
C_0	C_T

As you can see, the time subscript changes as we move along the time line. Of course we can indicate any point in time in-between t = 0 and t = T. Sometimes I will use a notation like t = t` (t prime) to show such a point.

One final note on the notation above: I clearly distinguish between the **value** and the **market price** for options, but not so much for the underlying stock. I do this because the focus now is on options. Stock values can also differ from market prices in an inefficient market, but for our purposes, we need not make that distinction.

Value of a Call at Expiration

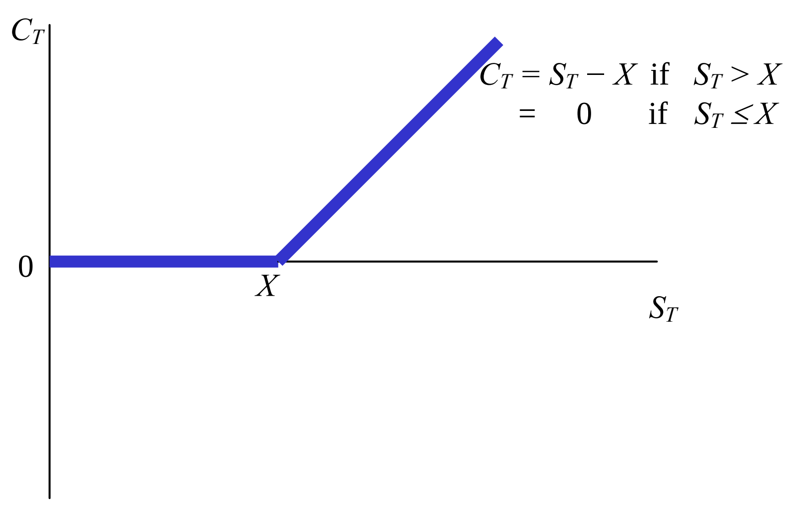
For each option there is a seller (writer) and a buyer (holder). We will first consider the value of the option to the holder. He will have the right but not the obligation to exercise. He will clearly exercise only if it is to his advantage to do so. Suppose we want to write down a function (or an equation) which gives us the value of the option at expiration to the option holder. If the stock price is greater than the strike price, the option holder would get more than he would pay, so he would exercise. Otherwise, he would just let that option **expire** without exercise. A function which describes this would look as follows:

$$C_T = S_T - X \quad \text{if} \quad S_T > X$$
$$C_T = 0 \quad \text{if} \quad S_T \leq X$$

The equation describes the call value at expiration as two "if statements". They are both dependent on the stock price at the expiration of the option. If the stock price is greater than the strike price, the stock you obtain by exercising is greater in value than the strike price you would give up. If the stock price is less than or equal to the strike price, you throw your option away, unexercised. Notice that only one of the if statements can be true for a given expiration day price.

The next learning module will cover how to move from a function such as above to a plot on a coordinate axes, where the *x* axis is the stock price at expiration and the y axis is the value of the call. It will build on the skills you will obtain from Lecture Notes #1 Part B. But for now, I have done all the work for you!

Value of Call to Holder at Expiration



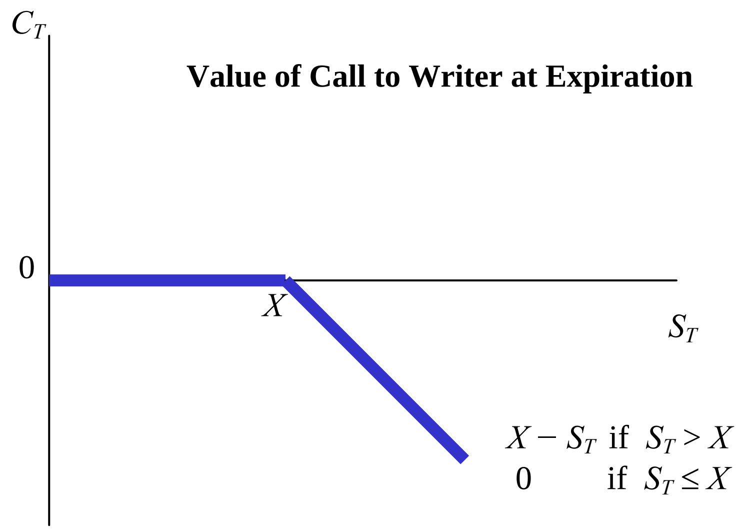
Nevertheless, you still can look at the end result, and be convinced that the above plot fits the equation.

Incidentally, your option ends up "**in-the-money**" if $S_T > X$, otherwise, it ends up "**out-of-the-money**".

What about the value of the call to the option writer? It is exactly the opposite of the holder. Since the writer has to give up the share, and receive the stock price, the cash flows should be reversed. Incidentally, if you consider money received as having a plus sign and money (or asset value) paid out as having a negative sign, you should have no problem creating the equations:

To the writer, the call value (or more appropriately, the liability) will be:

$$\begin{array}{ll} -S_T + X & \text{if} \quad S_T > X \\ 0 & \text{if} \quad S_T \le X \end{array}$$



Perhaps we can get a hint of why the buyer is called the option "holder". If you purchase the option, then at expiration you can benefit it the option ends up in-the-money. The call seller can only lose money at expiration, and he will do so if the call ends up in-the-money.

After the option is sold, only the buyer will benefit, and it will be at the expense of the writer! So it is to the advantage of the buyer to "hold" on to the option. My story about why the seller is called the "writer" is not so intuitive, but I believe the writer term comes from the fact that these contracts are instantaneously created (unlike shares of stock). If you think in terms of unique individual options as traded in over-the-counter markets, someone, like the seller, has to write the contract: "I give the holder of this contract the right to buy one share from me on or before such and such date for the price of $X."

Also, since only the holder will benefit from the option after it is sold, the holder must pay a price to the writer. This price is also called the option premium.

I will now go over an example of a call option using real data from the Chicago Board Options Exchange (CBOE). The data was taken from the Wall Street Journal (WSJ) print edition on 11/29/2000 which reports the previous day's (**11/28/2000)** closing price for a calls and puts with strike price = 45 on Cisco stock, with maturity for the following month (December).

			Call		Put	
Cisco	Strike	Expiration	Vol	Last	Vol	Last
51	45	December	232	8^{13}	1264	1^{25}

Some comments about the exchange-traded options trading on the CBOE in 2000, as well as how these prices were reported at that time in the WSJ.

The closing stock price for Cisco Systems on 11/28/2000 on Nasdaq, the stock's primary exchange, was $51.

Call contracts with a strike price of 45 and December expiration had a closing price (premium) of $8 1/8 or $8.125. The WSJ in 2000 rounded that up to 8.13 in anticipation of decimalization.

Exercise prices of options contracts are standardized. Example: 35, 40, 45, etc.

Expiration dates are also standardized. Options expire in selected months (1 to 8 months duration) on the Saturday following the third Friday of the month.

Cisco's December expiration is on Saturday, December 16th. Each contract gives the holder right to purchase 100 shares. The volume of trading for this particular call option was 232 contracts on 11/28/2000.

For the Cisco December 45 call option:

$S_0 = 51$ $\qquad X = 45$ $\qquad C^{MP} = 8.125$

What is the value of the call at expiration?

$$\begin{aligned} C_T &= S_T - 45 \quad \text{if} \quad S_T > 45 \\ &= \quad 0 \qquad\ \ \text{if} \quad S_T \le 45 \end{aligned}$$

Notice that you still can not get a numerical value, unless you know the stock price for Cisco at expiration. This is not available until the actual day, so options are financial assets with risk, just like the underlying stock. Further, option values will depend on the underlying stock's future price.

The terms in-the-money and out-of-the-money for calls can be applied not just at expiration but also anytime before expiration including the day the option is purchased. Instead of using $S_T < X$ and $S_T > X$, we would use $S_t < X$ and $S_t > X$. As you can see, the only difference is the subscript on the stock price, which now indicates day t rather than day T, the expiration day.

Is this call in- the- money <u>when purchased</u>?

For Cisco on 11/28/2000, the call is in-the-money since 51 > 45. Incidentally, if $S_t = X$ we call it "**at-the-money**".

You can also determine the **exercise value** of a call anytime. This is just the value of the call if we exercise it.

For calls, it would be:

$$\text{Exercise Value at time t} \quad \begin{aligned} &= S_t - X \quad \text{if} \quad S_t > X \\ &= \quad 0 \qquad \text{if} \quad S_t \leq X \end{aligned}$$

What is the exercise value of this Cisco call if we exercise it immediately after we purchase it?

$$\begin{aligned} &S_0 - 45 \quad \text{if} \quad S_t > 45 \\ &\quad 0 \qquad \text{if} \quad S_t \leq 45 \end{aligned}$$

At time of purchase, t = 0, and $S_0 = 51$. Since 51 > 45, the exercise value of the Cisco call at t = 0 is equal to 51 − 45 or $6.

Notice that 6 is less than the option premium 8.125!

Why are investors paying more for this option than its exercise value?

Investors did not buy the option just in order to immediately exercise it. They know there is some chance that it will become more valuable if they hold on to it for a while. Suppose the stock increase to a greater value? Of course, it could also decrease!

Often we break up the option price into two components: **Exercise Value** and **Time Value**. Other names for these components are: **Intrinsic Value** and **Speculative Value**.

After you obtain the exercise value, you can just subtract it from the option price to get the time value.

What is the exercise value and time value of the Cisco call?

We already obtained the exercise value above. The time value would be calculated as follows:

Option Price = Exercise Value + Time Value

8.125 = 6 + Time Value

Time Value = 2.125

As you can see, we can back out the time value once we calculate the exercise value. Both the exercise value and the time value must be a non-negative value in a rational market.

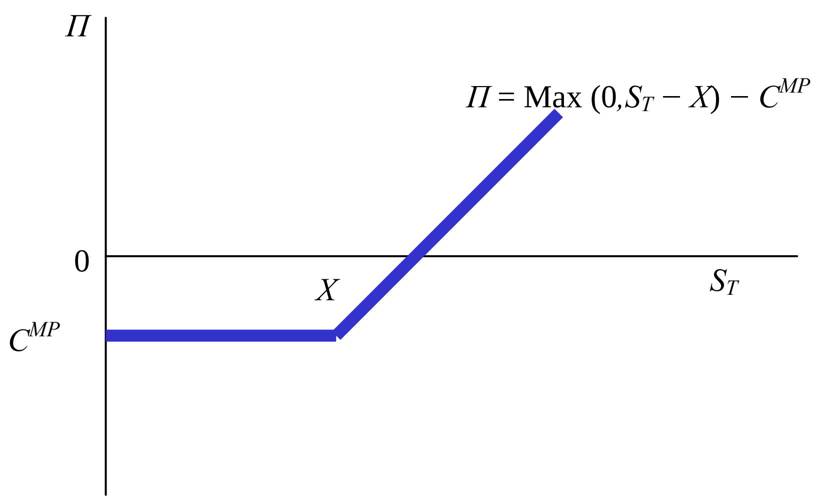
Profit of a Call at Expiration

A **profit diagram** at option expiration is similar to the option value diagram, but it also includes the premium paid for the call. For reasons of simplification, I ignore the issue of different values of cash flows due to it occurring in different points in time. That is, in finance we usually discount future cash flows in to present value. We will also do this in this class but make this single exception for profit diagrams.

In general it is easy to modify the value of the call to the holder at expiration to become the **profit on a call to the holder at expiration**:

$$\begin{aligned} \Pi &= S_T - X - C^{MP} \quad \text{if} \quad S_t > X \\ &= -C^{MP} \quad \text{if} \quad S_t \leq X \end{aligned}$$

Profit of Call to Holder at Expiration



Notice that the y axis now has the Greek letter pi to indicate profit. Also, compared to the previous value diagram, it seems that it is identical but everything is lower by the amount of the call premium.
Perhaps you noticed a different way of writing the profit of a call inside the diagram! You can write the value and the profit either with the use of two "if" statements:

$$C_T = S_T - X \quad \text{if} \quad S_T > X$$
$$C_T = \quad 0 \qquad \text{if} \quad S_T \leq X$$

or with the use of a max function as follows:

$$C_T = \text{Max}(0,\ S_T - X)$$

where Max(a,b) = a if a > b
= b if b > a

As you can see the use of a max function allows you to write two if statements in a more compact manner. While a max function is easier to write, I would recommend working with the two if statements when it comes to manipulating the equations that will be required when we start to combine derivatives. The use of the max does allow us to write the profit to a call holder in two identical ways:

$$\Pi = S_T - \text{X} - C^{MP} \quad \text{if} \quad S_t > \text{X} \qquad \text{or} \qquad \Pi = \text{Max}\ (0,\ S_T - \text{X}) - C^{MP}$$
$$= \ - C^{MP} \qquad \text{if} \quad S_t \leq \text{X}$$

On an organized exchange like the CBOE, an option contract generally gives the right to buy 100 shares of the underlying stock, and the premium is also 100 times the indicated market price.

The profit on a **call contract**, if held until expiration will be:

100 x [Max ($0,\ S_T - X$) $- C^{MP}$]

What is the profit on a contract of the above Cisco call, if the stock price is $60 on expiration day?

100 x [60 − 45 − 8.125] = **$687.50**

What will be the profit if the stock price at expiration ends up at $44*?*

100 x [− 8.125] = **− $812.50**

As you can see, in the first instance, the call finished in-the-money while in the second example it finished out-of-the-money.

This whole section is on the profit to the call holder, but you can do both the diagram as well as calculate profits given a specific ending stock price for the **call writer**. The profit will be identical in magnitude but opposite in sign; the diagram will also reflect the nature of this result.

Value and Profit to Put Holders

In this final section of this lecture, I will repeat for the put holder what we have done above for the call holder.

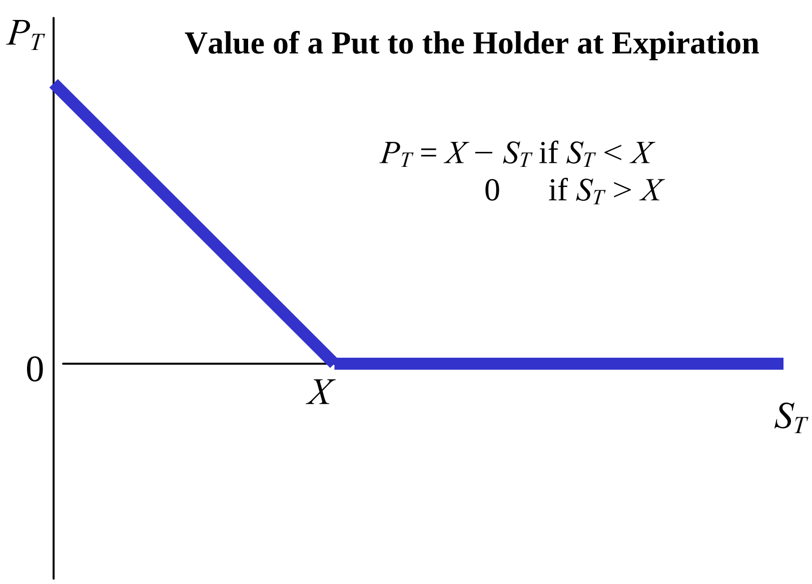
The put gives the holder the **right to sell** the underlying stock. As with the call, there is a put holder (the buyer of the option) and a put writer (the seller). The value to the holder at expiration will be as follows:

$$\begin{aligned} P_T &= X - S_T \text{ if } S_T < X \\ &= 0 \quad \text{if } S_T \geq X \end{aligned}$$

or

$$P_T = \text{Max}(0, X - S_T)$$

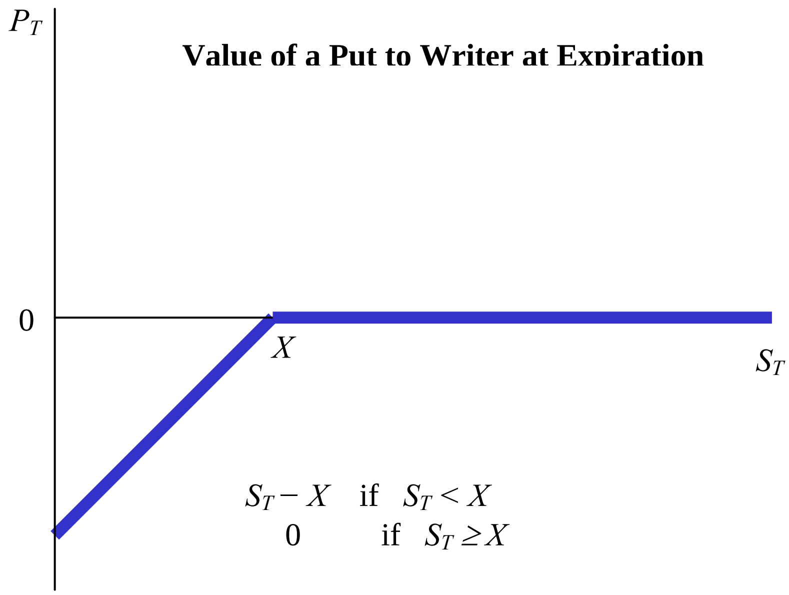
A put is in-the-money when S < X and out of the money when S > X. This is the opposite of a call. In general, the value of a put to the holder will look like this:



If we want to consider the value (or more correctly, the liability) from the put writer's perspective:

$$
\begin{array}{ll}
S_T - X & \text{if } S_T < X \\
0 & \text{if } S_T \geq X
\end{array}
$$

We can also plot this function:



Let's consider the Cisco December 45 put option as described in the WSJ above:

$S_0 = 51$ $\quad\quad$ $X = 45$ $\quad\quad$ $P^{MP} = 1.25$

Is this put in-the-money when purchased?

Since the stock price is 51, and 51 > 45 = X, it is out-of-the-money. Don't forget that this is opposite of how in-the-money/out-of-the-money works for the call.

What is the exercise value and the time value of the Cisco put immediately after the purchase?

Exercise Value of a put at anytime is:

$$
\begin{array}{ll}
X - St & \text{if } S_t < X \\
0 & \text{if } S_t \geq X
\end{array}
$$

so for the Cisco put, Exercise Value = 0

Option Price = Exercise Value + Time Value

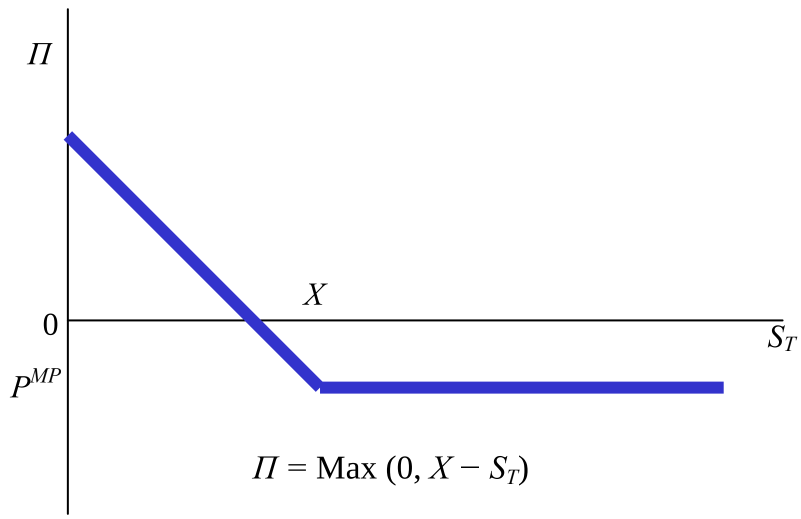
$$1.25 = 0 + \text{Time Value}$$

$$\text{Time Value} = 1.25$$

The whole value of the put is due to the time value.

In general, the profit on a put contract could be plotted as:

Profit of Put to Holder at Expiration



What is the profit on a contract of the above Cisco put, if the stock price is $60 on expiration day?

$$100\ [\text{Max}\ (0, X - S_T) - P^{MP}]$$

$$100\ [\ \text{Max}(0, 45 - 60) - 1.25] = 100\ [\text{Max}(0, -15) - 1.25]$$
$$100\ [0 - 1.25] = 100\ [-1.25] = \mathbf{-\$125}$$

What will be the profit if the stock price at expiration ends up at $44*?*

$$100\ [\text{Max}(0, 45 - 44) - 1.25] = 100\ [\text{Max}(0, 1) - 1.25]$$
$$100\ [1 - 1.25] = 100\ [-.25] = \mathbf{-\ \$25}$$

Notice that in the first example, the option is out-of-the-money, and you would not exercise, but in the second example, it is in-the-money, and you would exercise but you still end up with a loss! You are essentially in-the-money but below the break-even point for this option. But if you would not exercise, you would lose even more money.